

# Keimbildung

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Physikalische Chemie  
Universität zu Köln

Dr. Judith Wölk



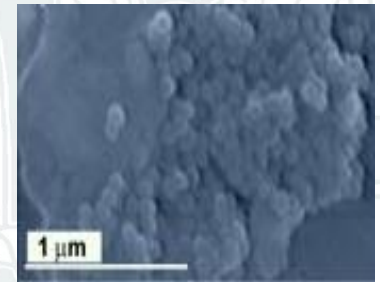
# Motivation

- **Atmosphärische Aerosol Bildung**



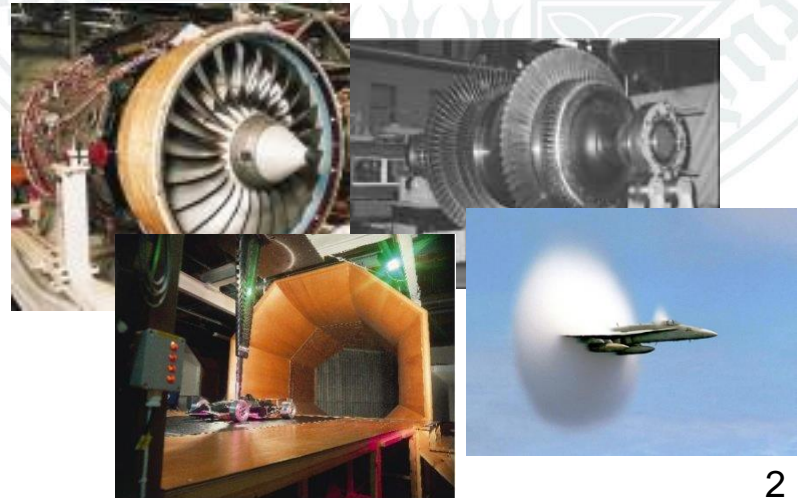
- **Partikel Herstellung:**

- Bildung von Nanokristallen
- Herstellung von qualitativ hochwertigen Pudern
- Medizin: Bildung von Gallensteinen



- **Kontrolle der Keimbildung in**

- Turbinen, Dampfturbinen
- Reaktionsgefäßen
- Windtunneln
- Düsenflugzeugen



# Motivation



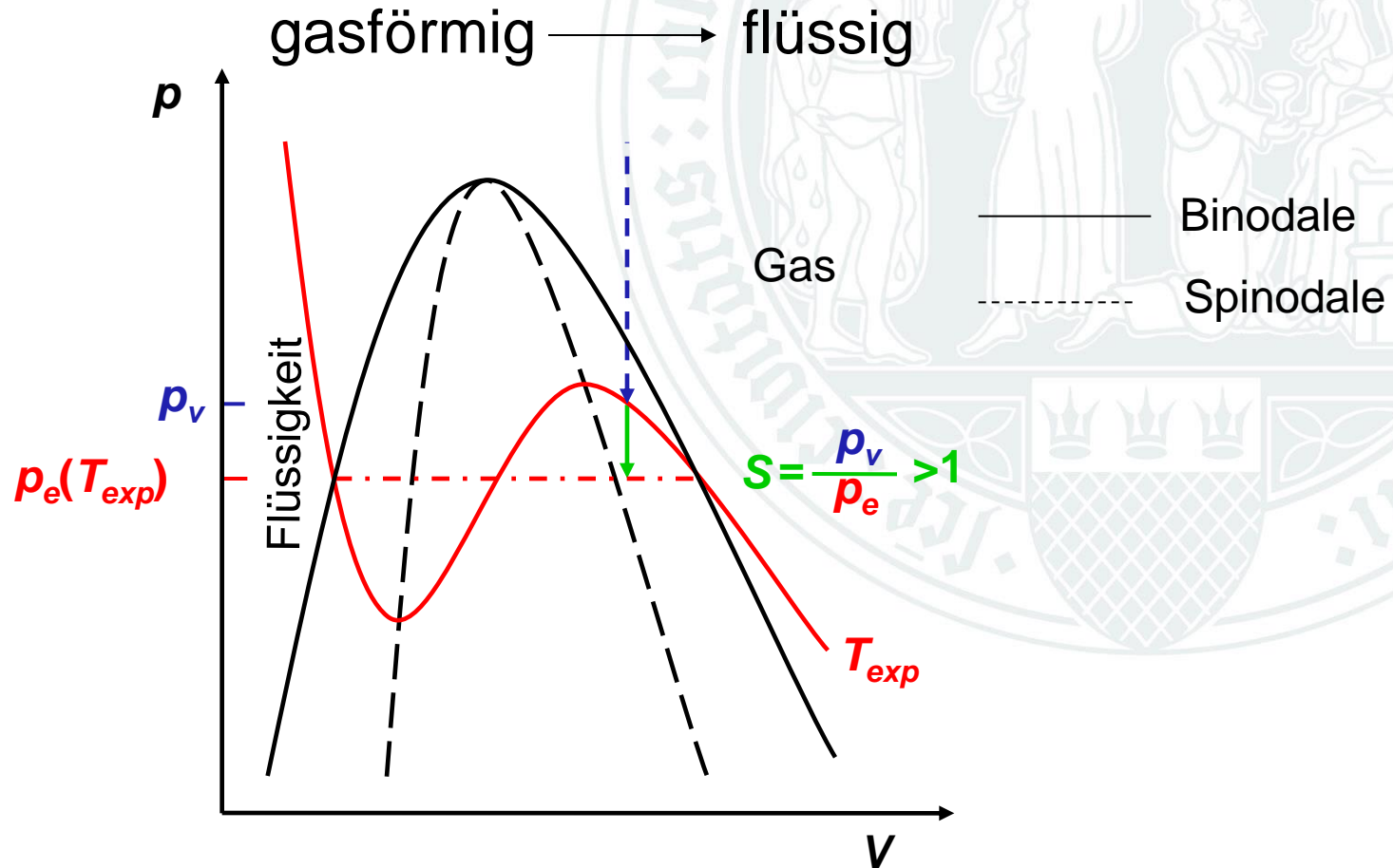
# Grundlagen

- **Keim:**  
**Kleiner Körper, einer neuen Phase, der sich mit der ihn umgebenden Mutterphase im labilen Gleichgewicht befindet.**



# Grundlagen

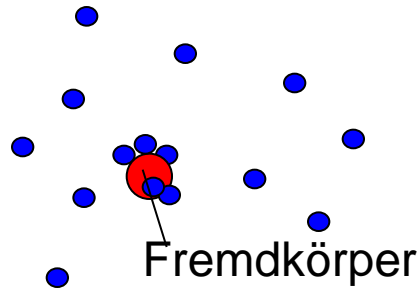
## Keimbildung: Phasenübergang erster Ordnung





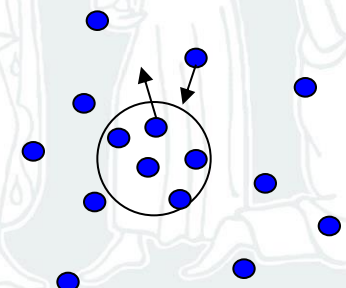
# Grundlagen

## Heterogene Keimbildung

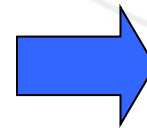


Keime, die durch Anlagerung der sich ausscheidenden Substanz an einem Fremdkörper entstehen

## Homogene Keimbildung

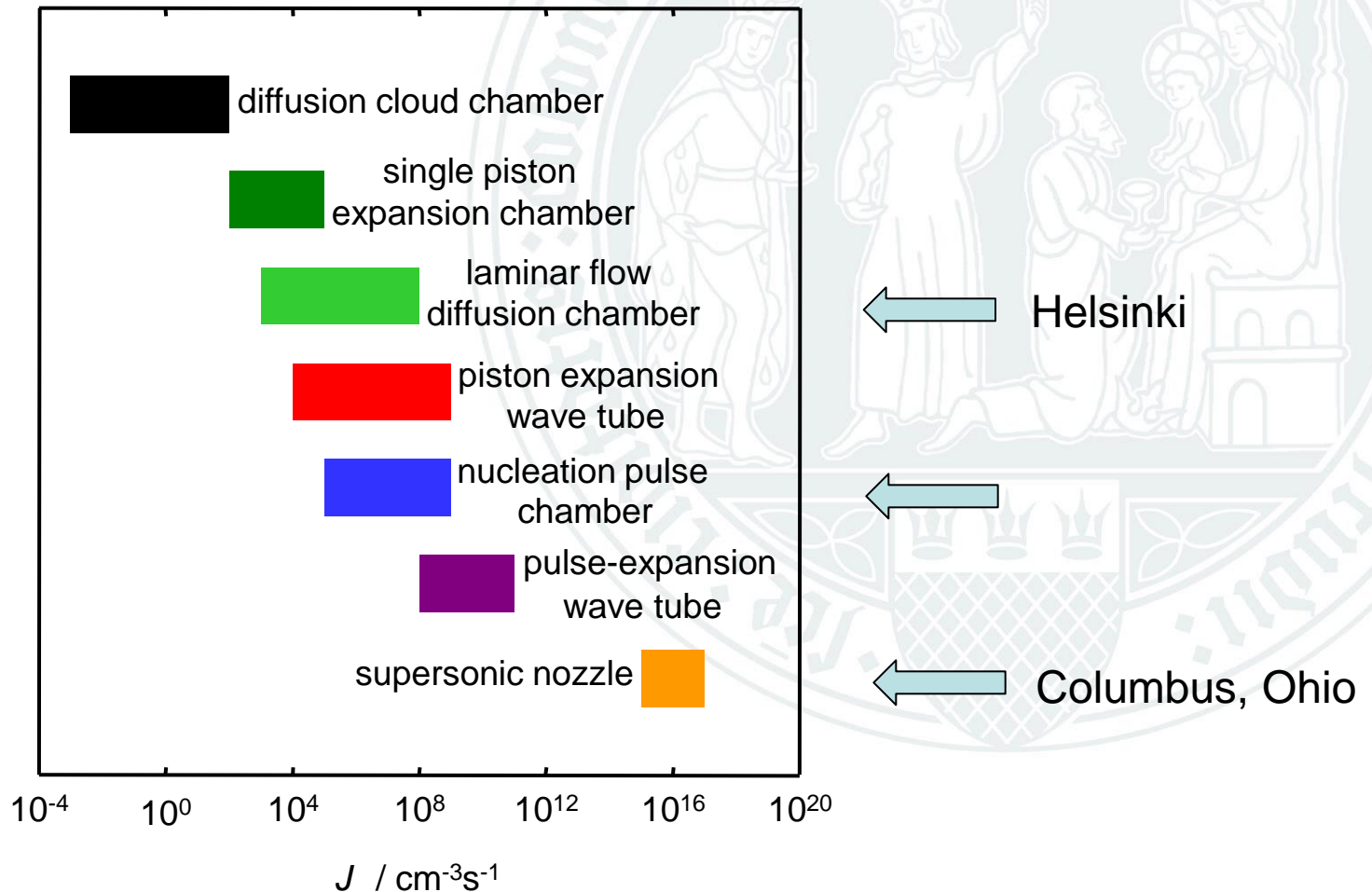


Keime, die durch eine vielstufige homogene Reaktion entstehen.



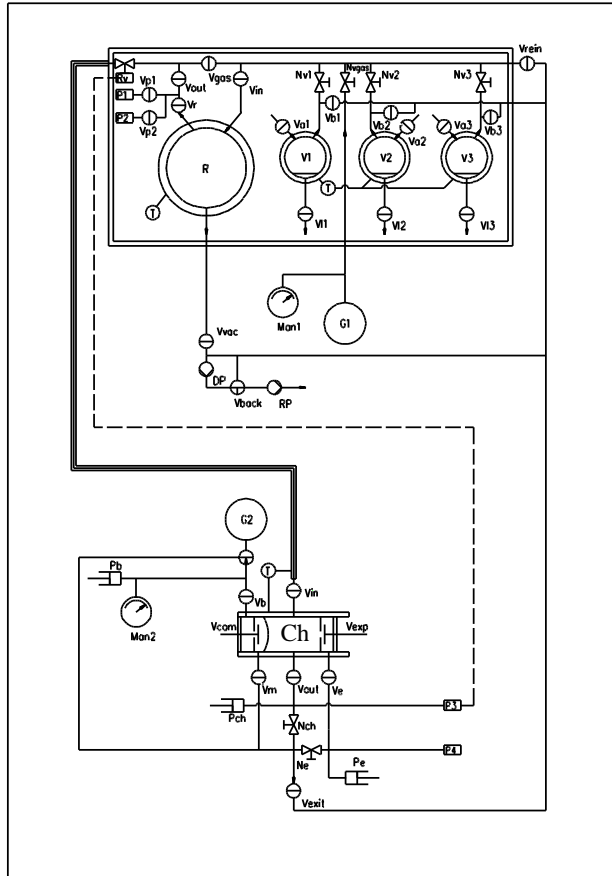
- Wasser
- Alkohole
- Argon
-

# Experimentelle Methoden



Keimbildungsrate: Zahl der Teilchen die pro  $\text{cm}^{-3}$  und Sekunde gebildet werden

# Nukleationspulskammer

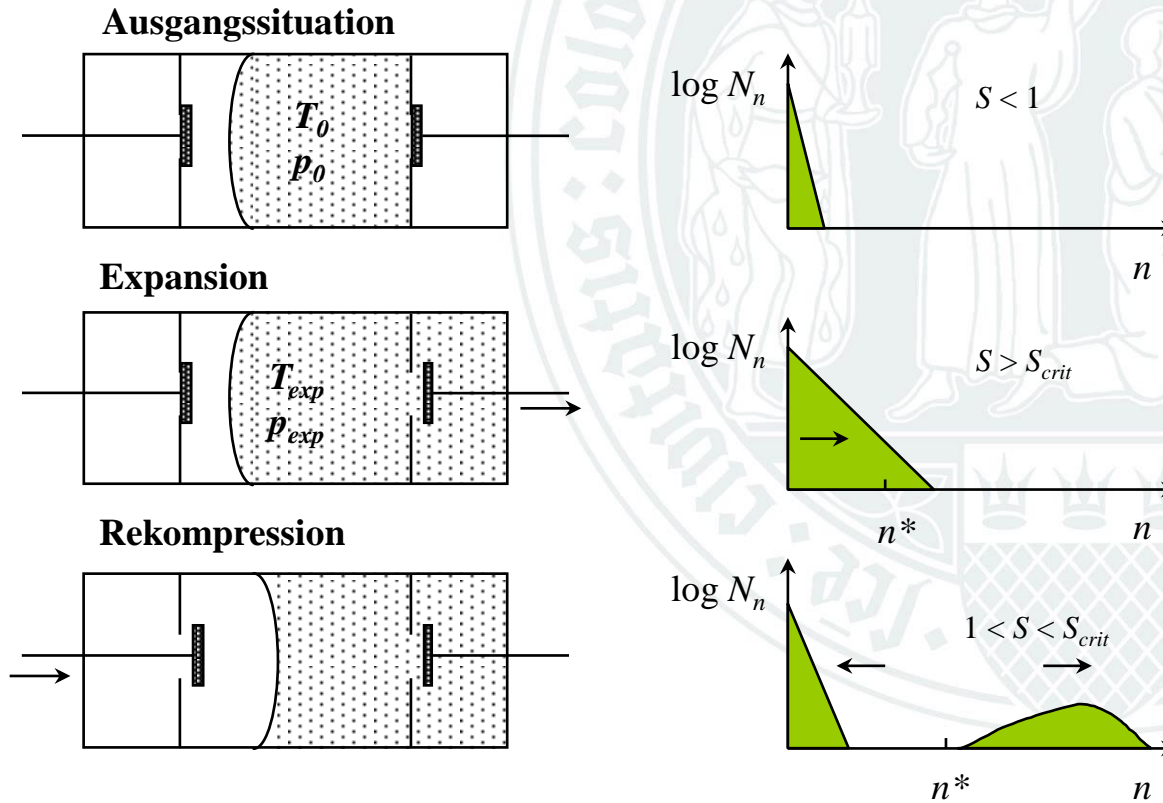


V1, V2, V3	Verdampfer
P1, P2, P3, P4	MKS Baratrons
R	Mischkolben
$N_{Vi}$	Nadelventile
$V_i$	Absperrventile
$P_i$	Regelvolumina
G1	Argongasflasche
G2	Stickstoffgasflasche
T	Thermostaten
$R_v$	elektronisches Regelventil
Man1, Man2	analoge Manometer
Ch	Nukleationspulskammer
-----	elektronische Regelleitung

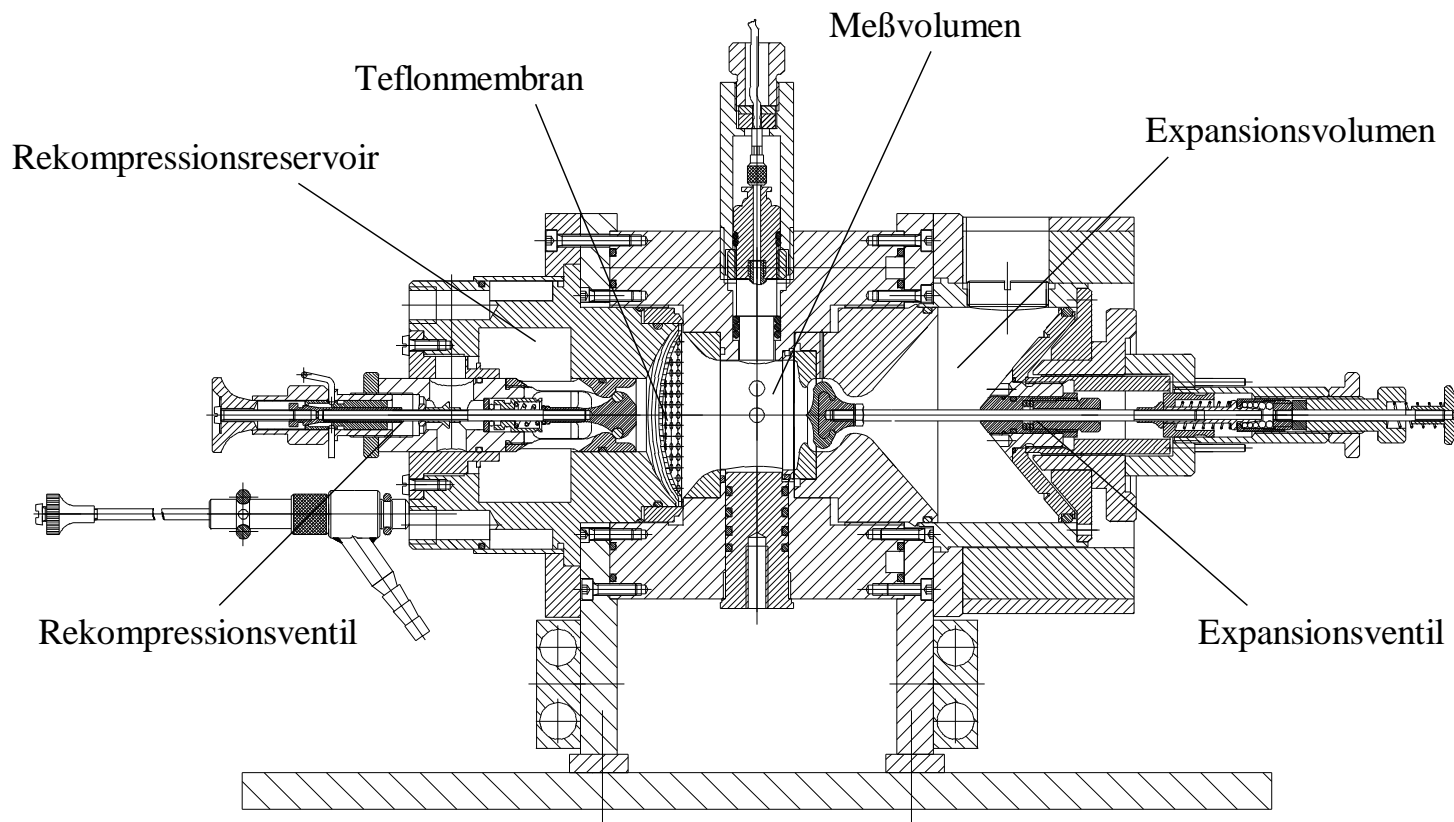
Schematische Darstellung der Messapparatur



# Nukleationspulskammer

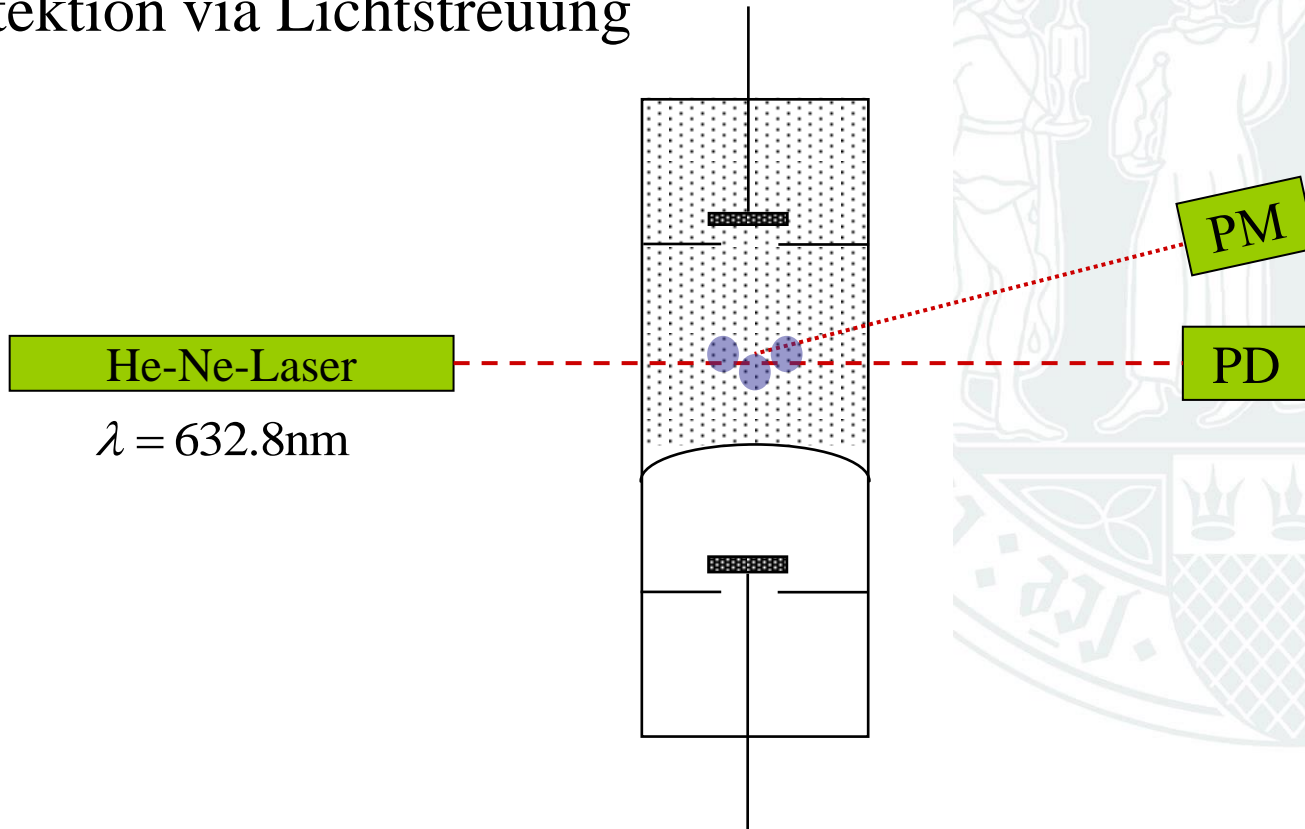


# Nukleationspulskammer



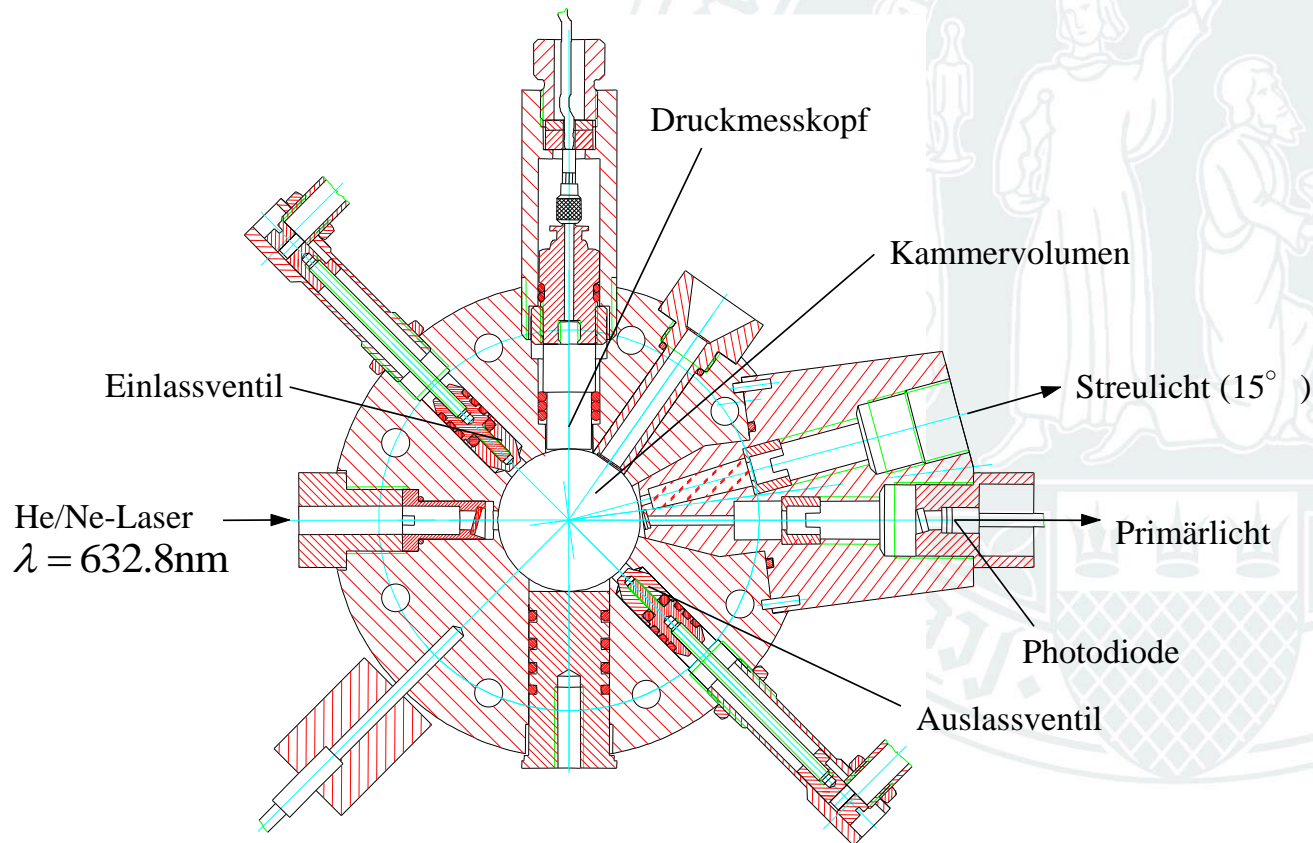
# Nukleationspulskammer

Detektion via Lichtstreuung

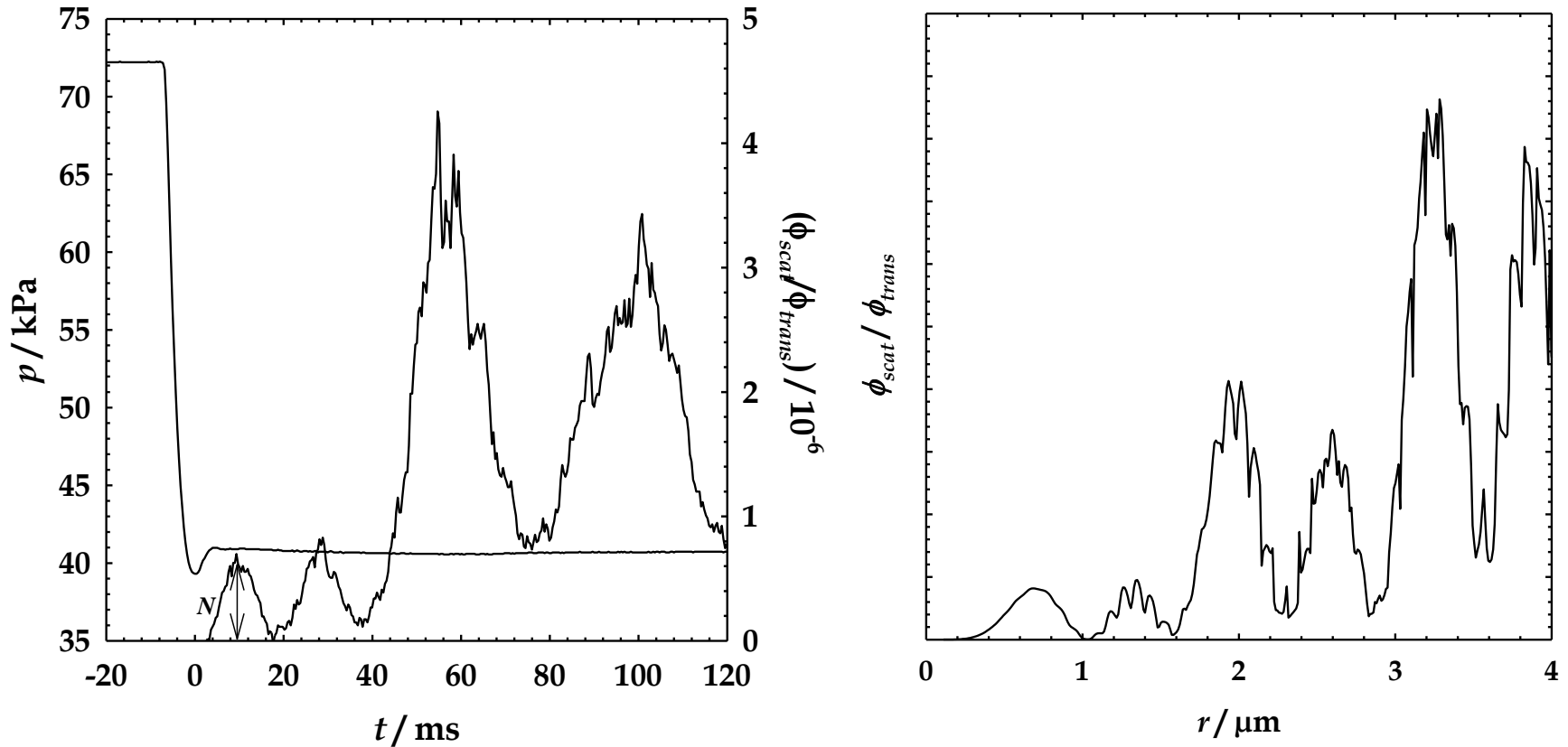


**Mie-Streuung:** Streuung an sphärischen Objekten, deren Durchmesser in etwa der Wellenlänge der Strahlung entspricht.

# Nukleationpulschammer

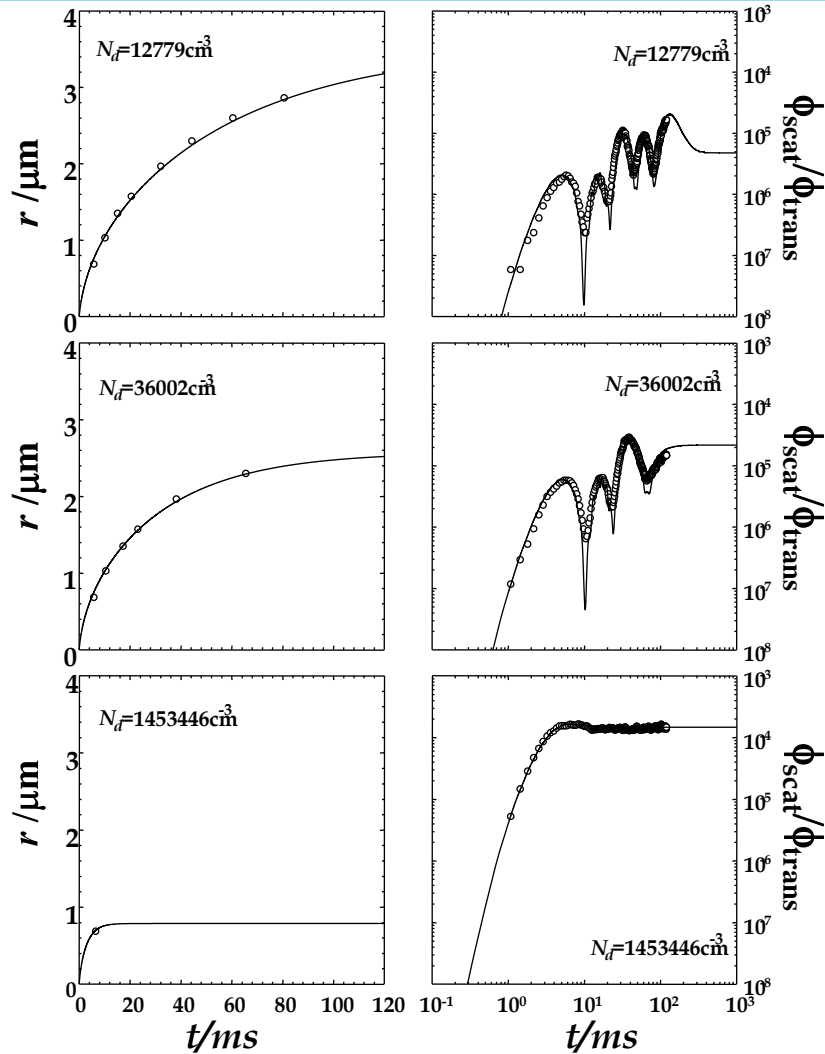


# Nukleationspulskammer

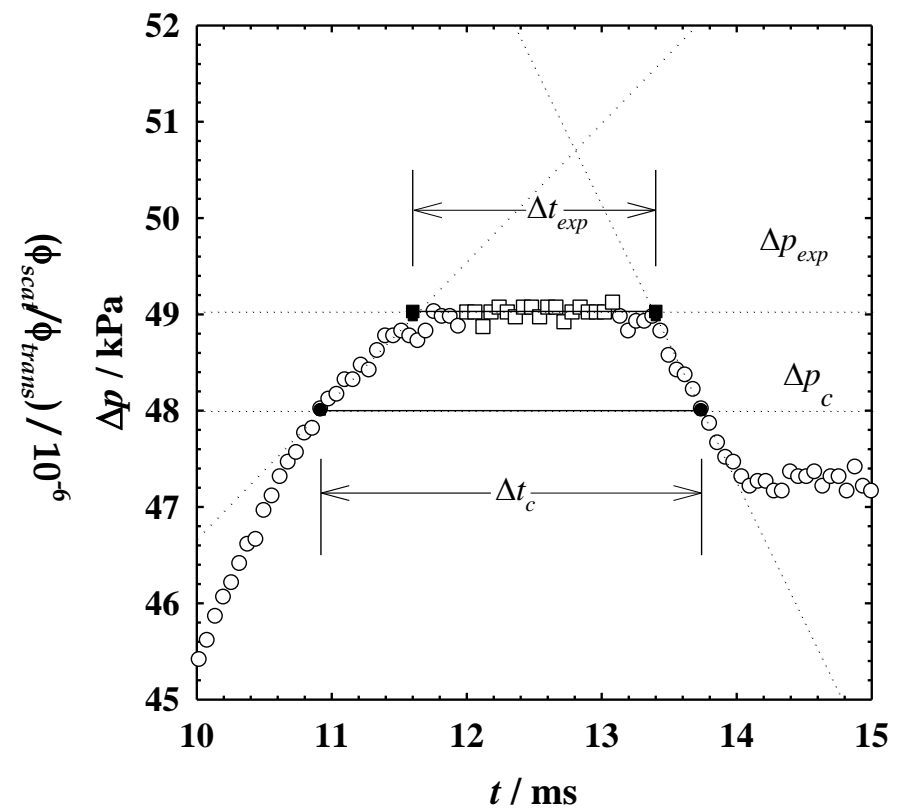
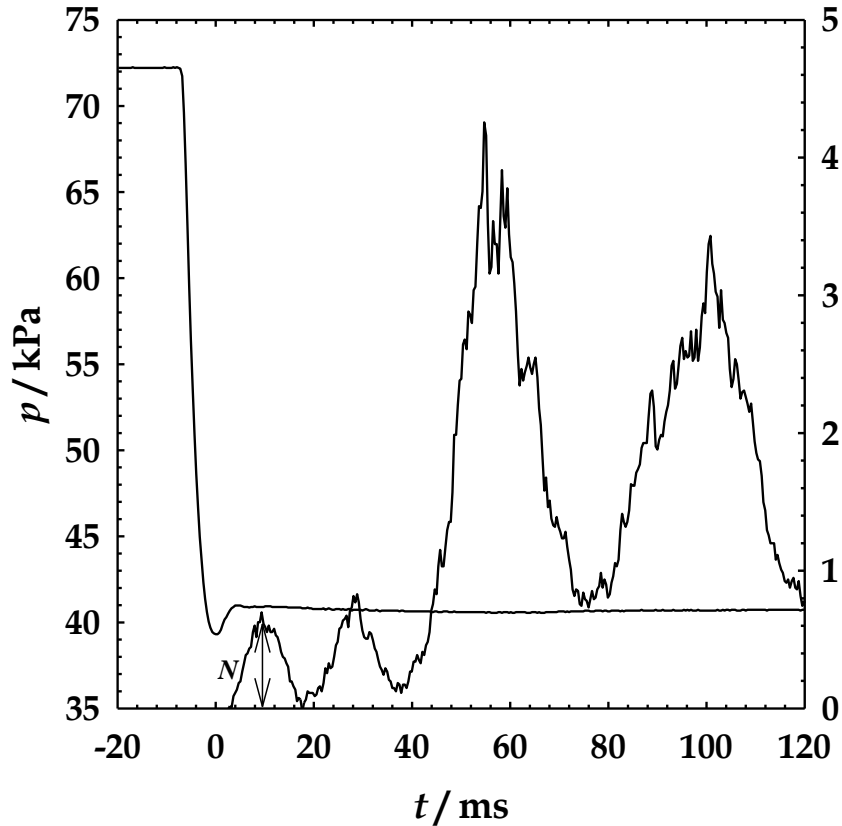




# Nukleationspulskammer



# Nukleationspulskammer



Höhe des ersten Mie-Maximums  $\Rightarrow$  Teilchenzahldichte  $N$   
 Pulsevaluierung  $\Rightarrow$  Pulslänge  $\Delta t \approx 1$  ms

$$J = \frac{N}{\Delta t}$$

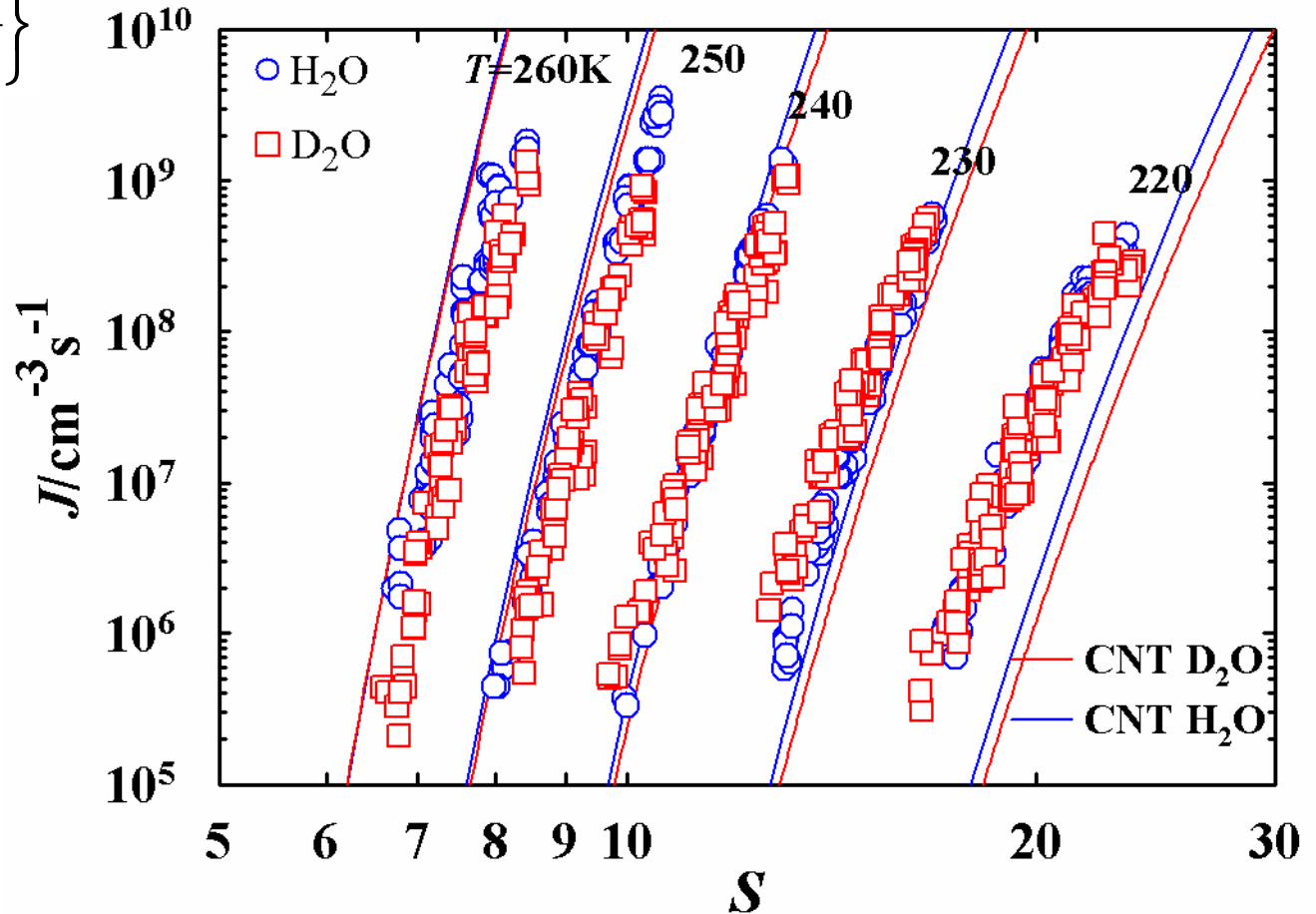
15



# Keimbildungsraten / Wasser

$$J_{\text{CNT}} = K \exp\left\{-\frac{\Delta G^*}{kT}\right\}$$

$K$  = kinetic prefactor  
 $\Delta G^*$  = work of formation



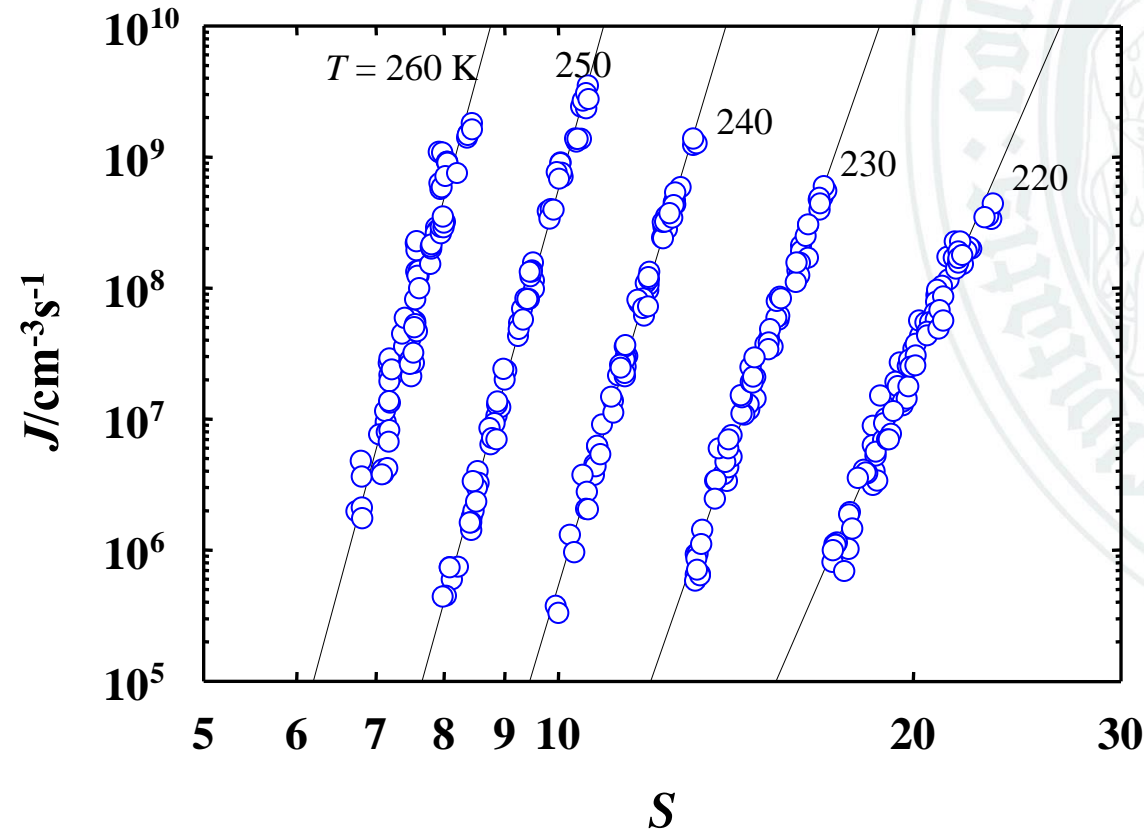
J. Wölk, R. Strey, *J. Phys. Chem.* **105**, 11683, (2001)

Becker, R., and Döring, W., *Ann. Phys.* **24**, pp. 719 (1935).



# Keimbildungsraten / Wasser

H<sub>2</sub>O-Argon

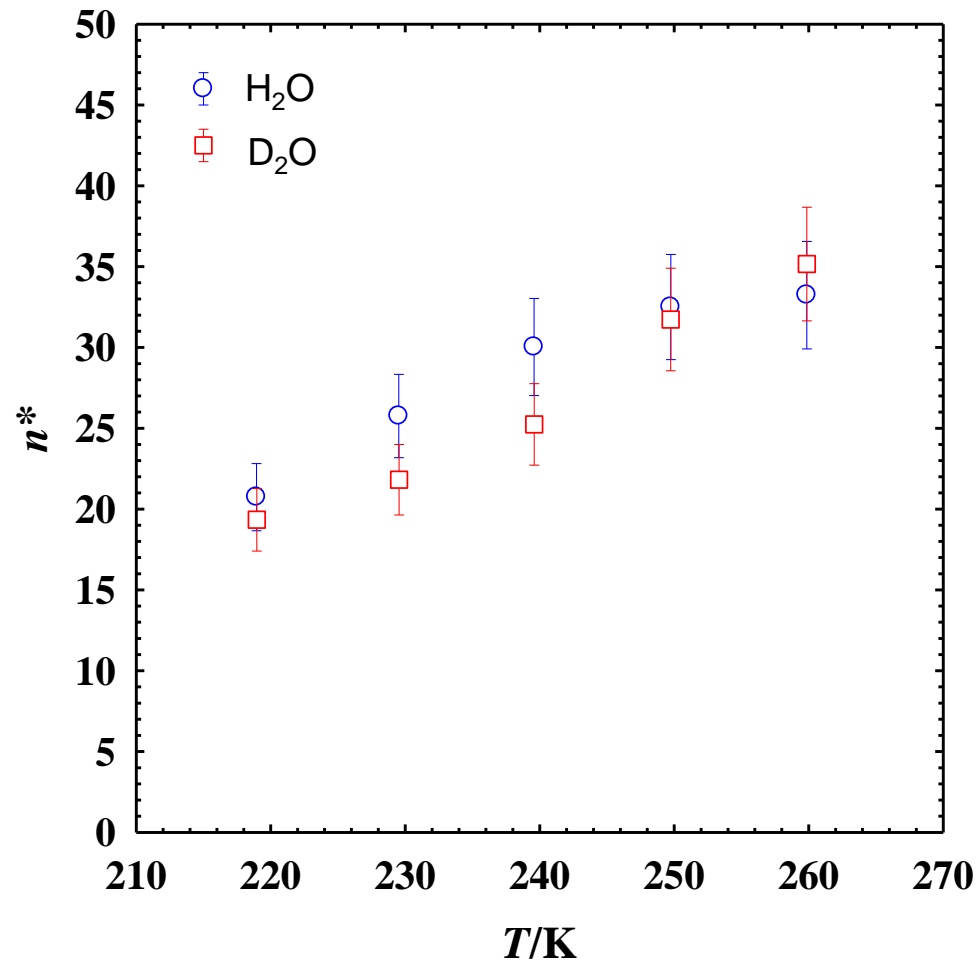


Kashchiev-Theorem:

$$\left. \frac{\partial \ln J}{\partial \ln S} \right|_T \cong n^*$$



# Kritische Clustergröße





# Laminar flow diffusion chamber - LFDC

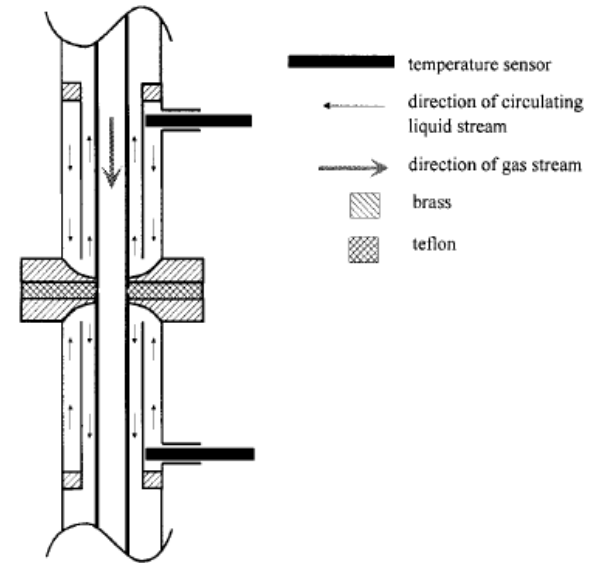
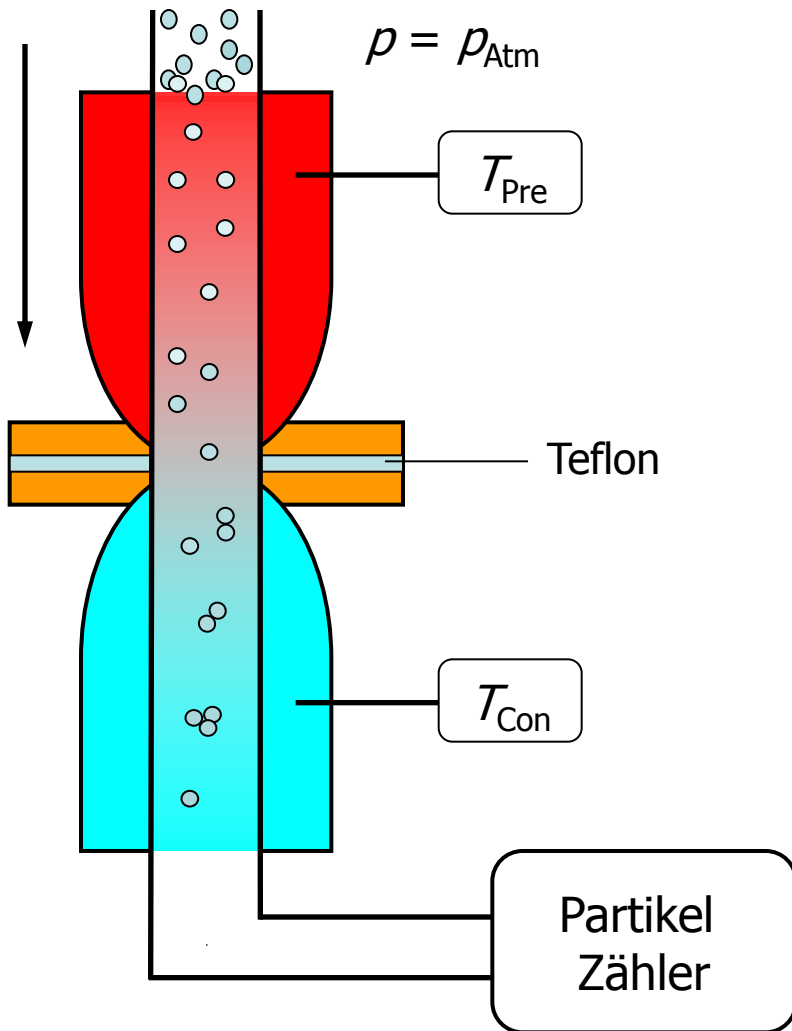


Figure 2. Schematic cross section of the connection between preheater and the condenser.

- **Partikel Zähler:**

→ **Zahl der Teilchen /Zeit**  $N^* = \frac{n}{\Delta t}$

- $\Delta V$  → nicht definiert

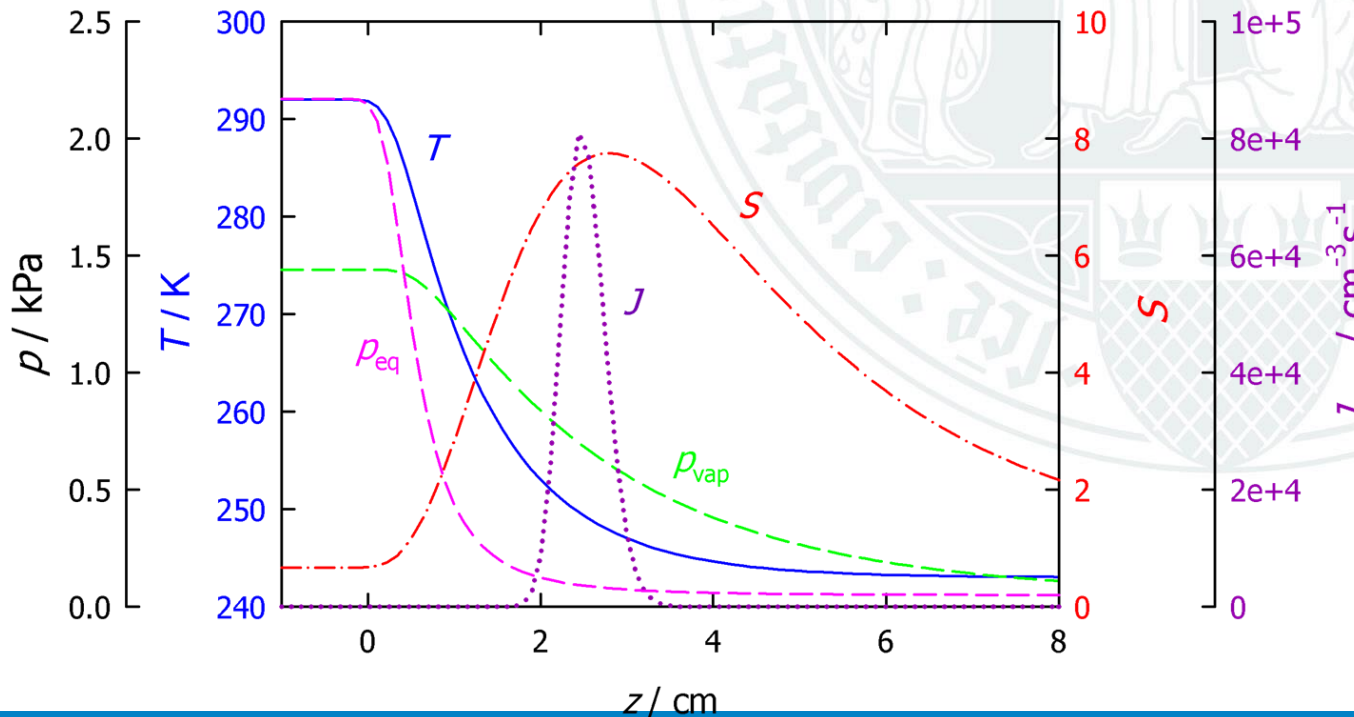
# LFDC: experimentelle Keimbildungsrate

$$J = \frac{n}{\Delta t \Delta V}$$

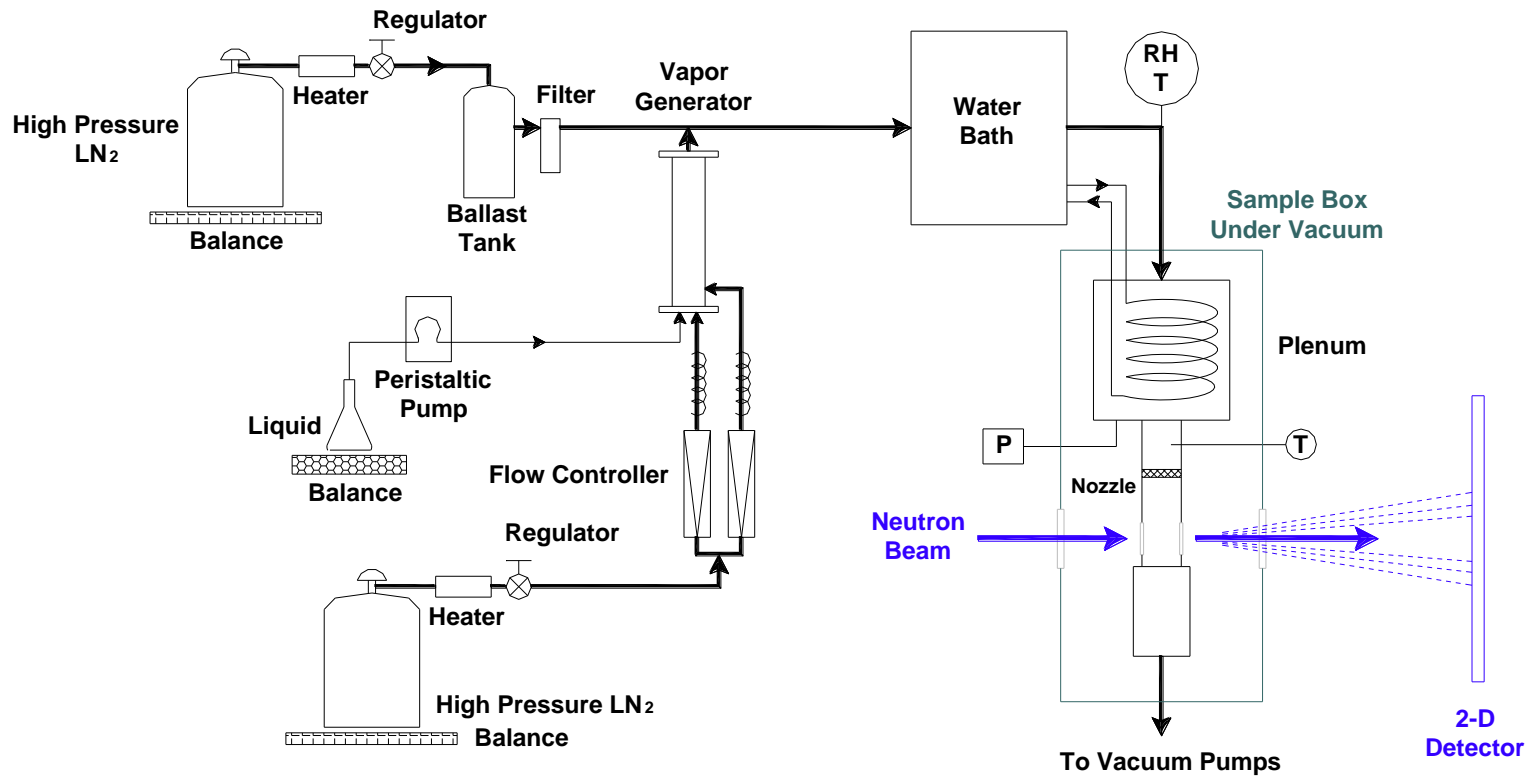
$$\frac{J_{\text{exp}}^{\text{max}}}{\int J_{\text{exp}} dV} = \frac{J_{\text{theo}}^{\text{max}}}{\int J_{\text{theo}} dV}$$

$$J_{\text{exp}}^{\text{max}} = \frac{1}{\Delta V} \int J_{\text{theo}} dV \int J_{\text{exp}} dV$$

$$N^* = \frac{n}{\Delta T}$$



# Überschalldüse



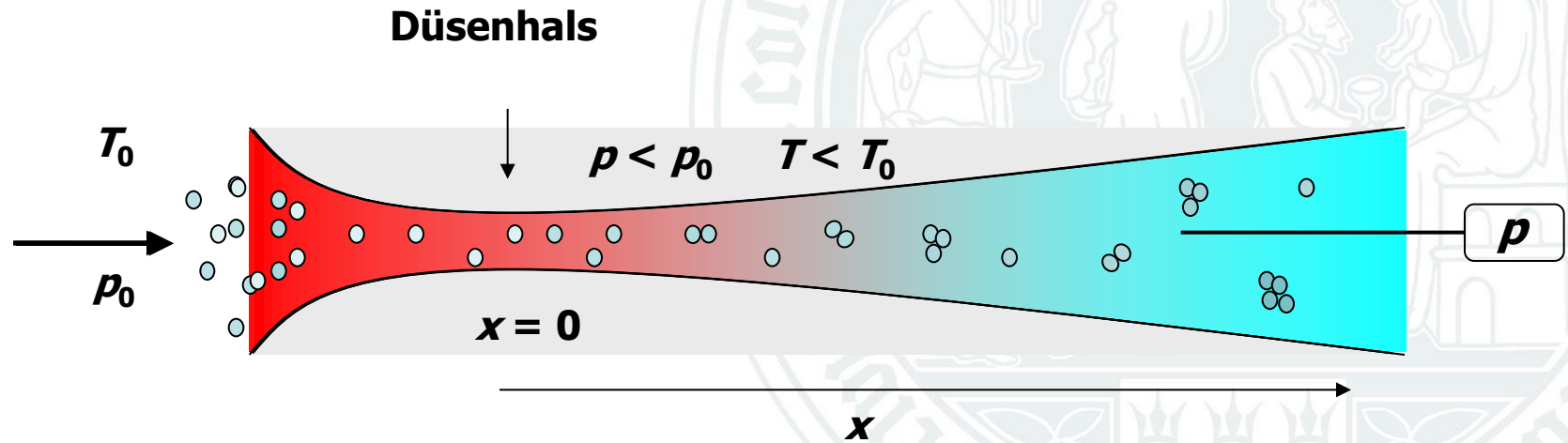
P - Pressure    T - Temperature    RH - Relative Humidity

 - Heating Tape

# Überschalldüse

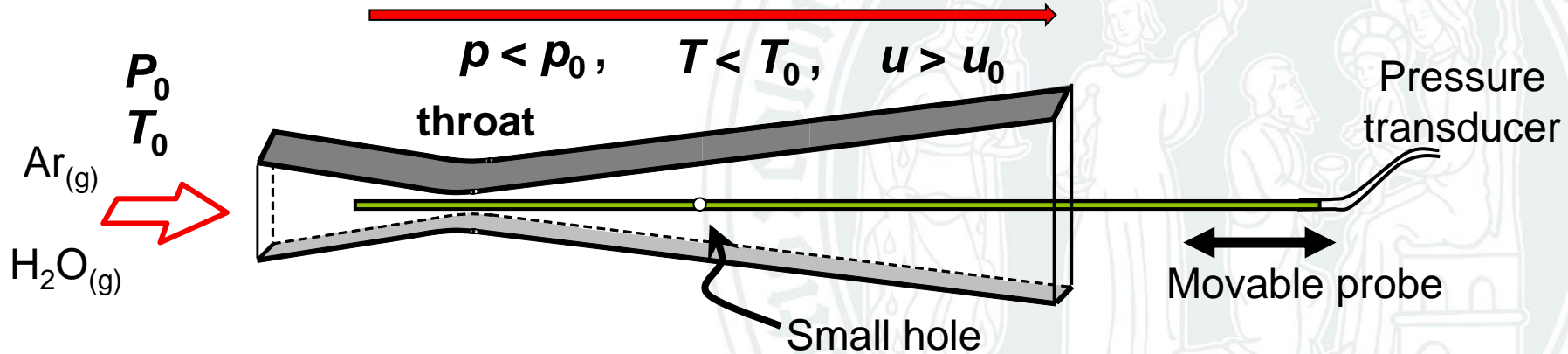


# Überschalldüse





# Überschalldüse



... Lots of carrier gas; <2% condensible

$u \sim 400 \text{ m/s}$

$\Delta x/u = \Delta t$

$1 \text{ mm} \sim 2.5 \mu\text{s}$

## gemessen

- Anfangsdruck
- Anfangstemperatur
- Gasphasenzusammensetzung

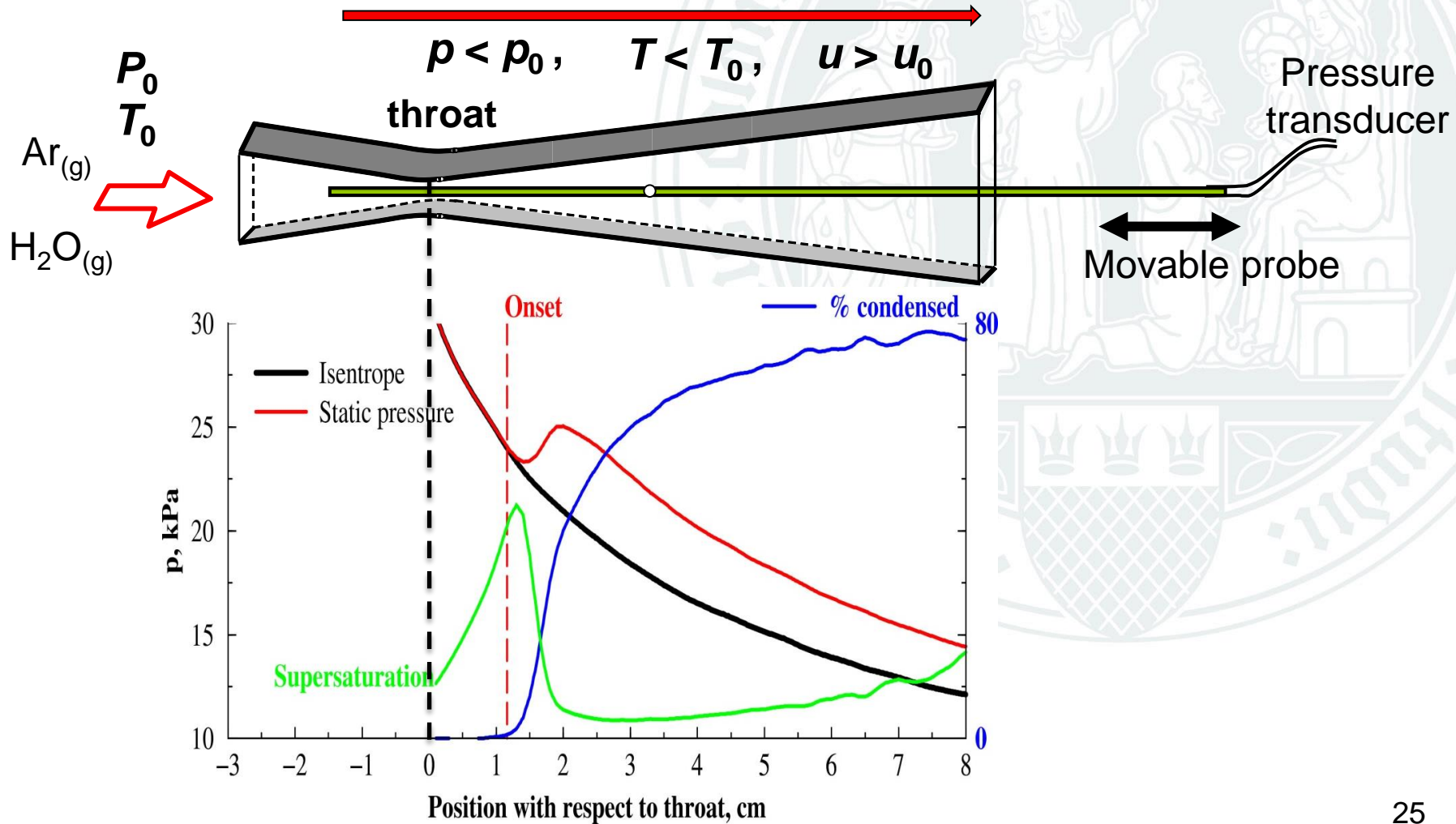
## gemessen

- statischer Druck

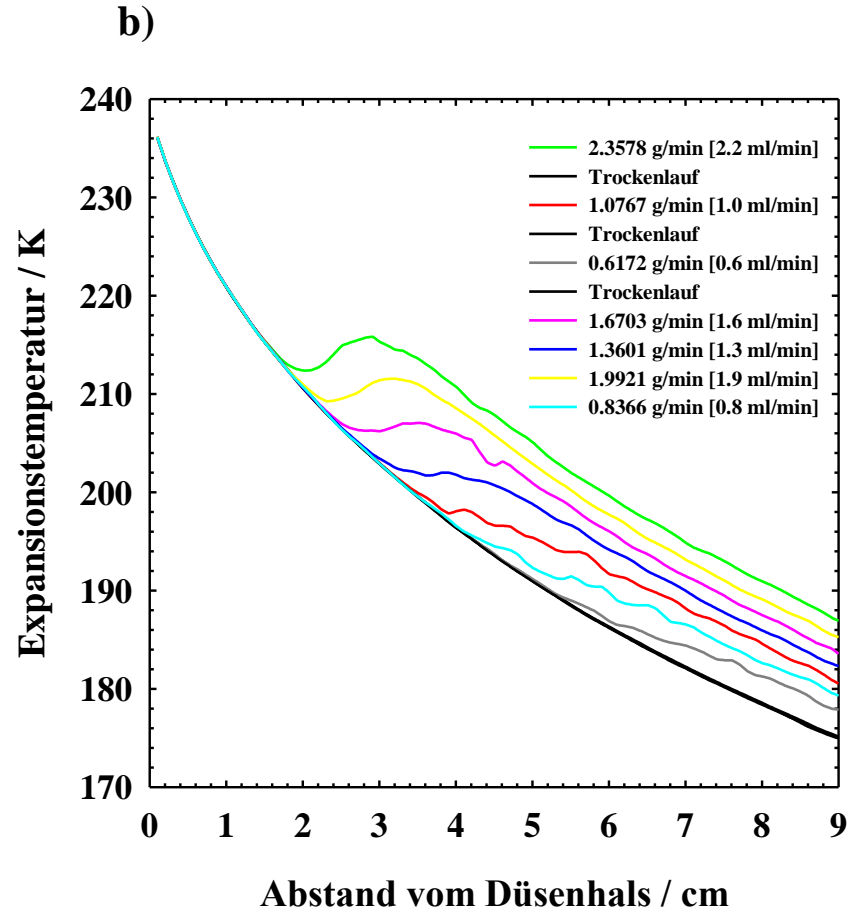
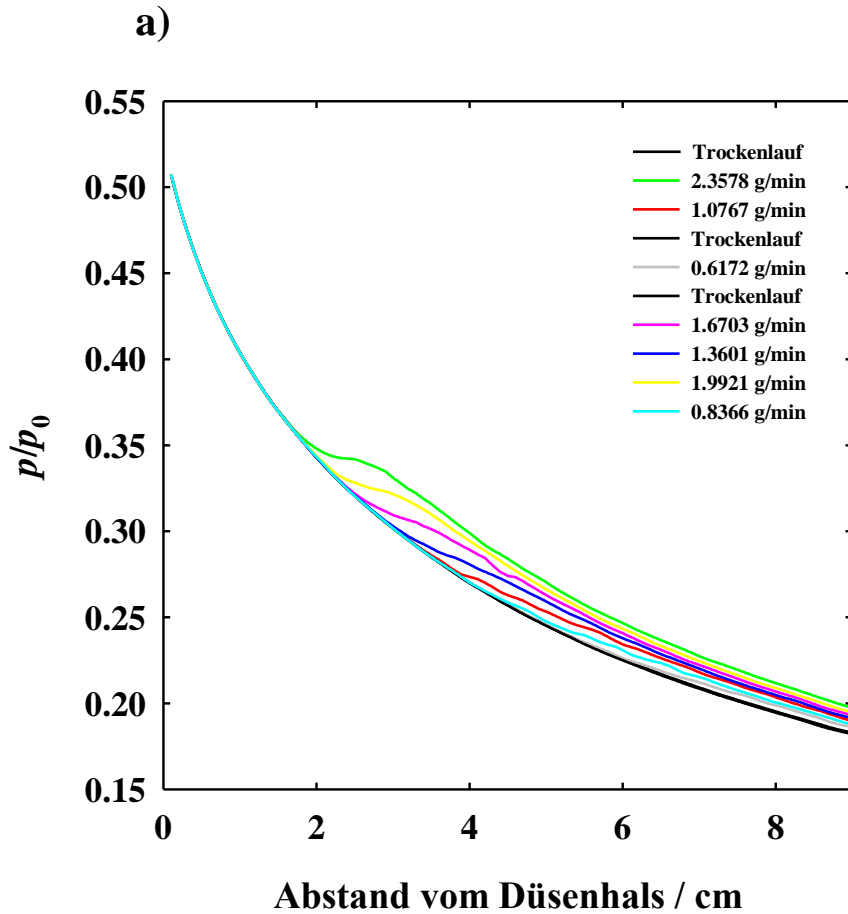
## Berechnet:

- Kondensationsmenge

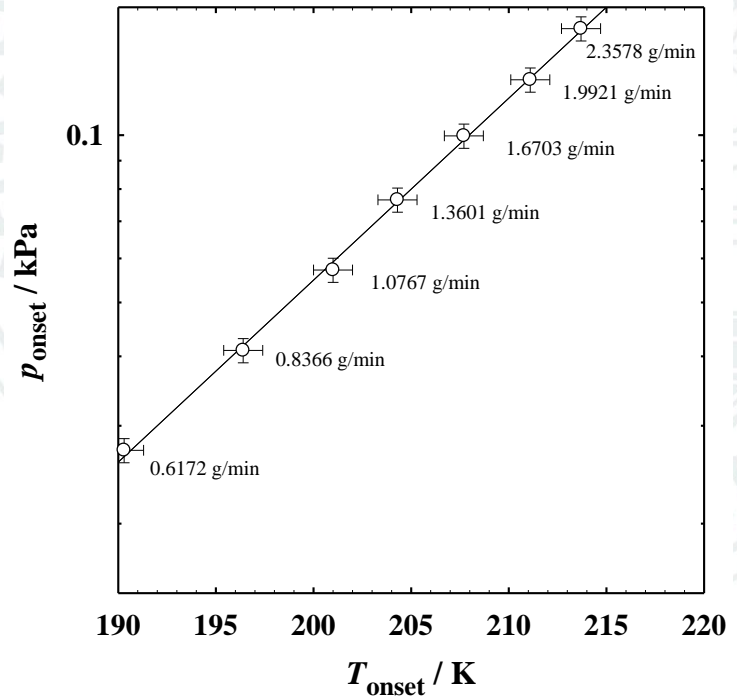
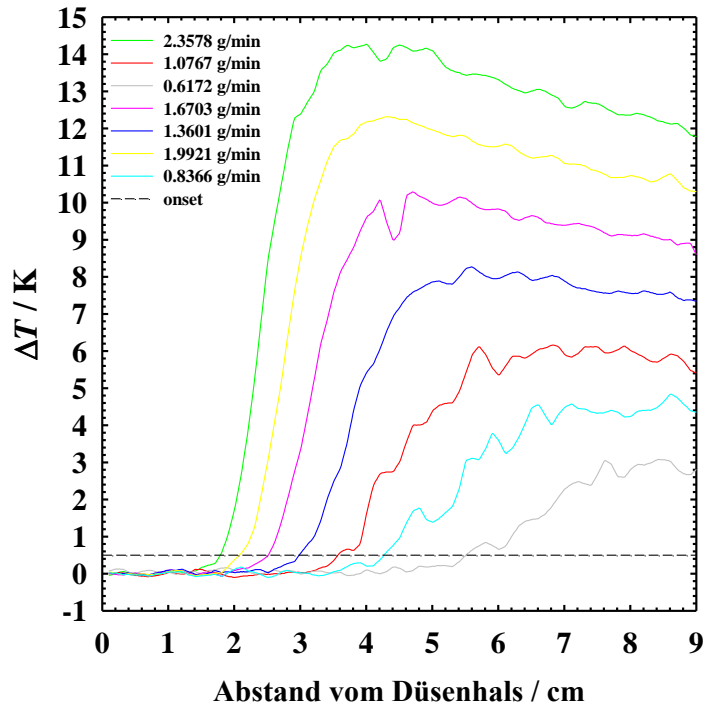
# Supersonic Nozzle



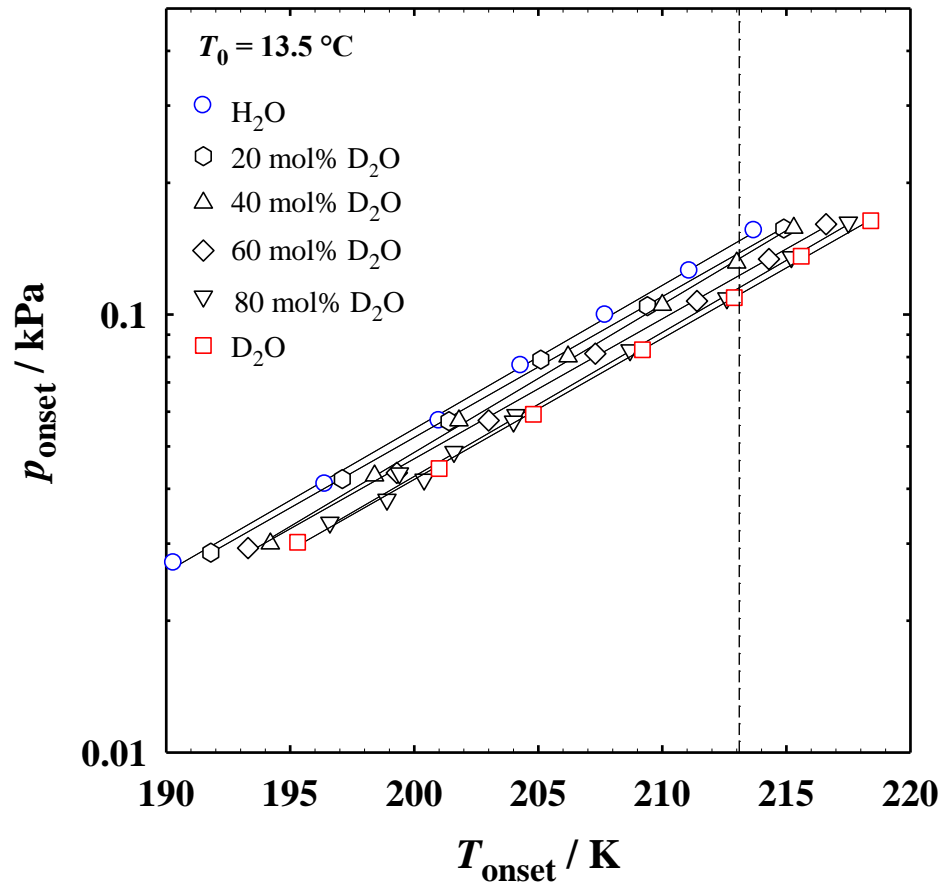
# Überschalldüse



# Überschalldüse

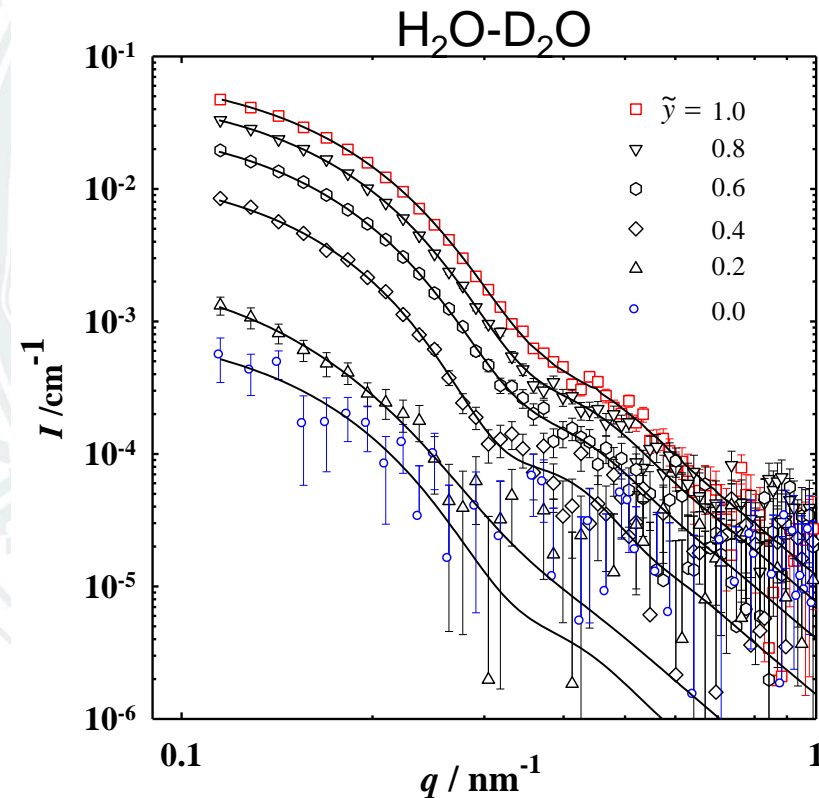
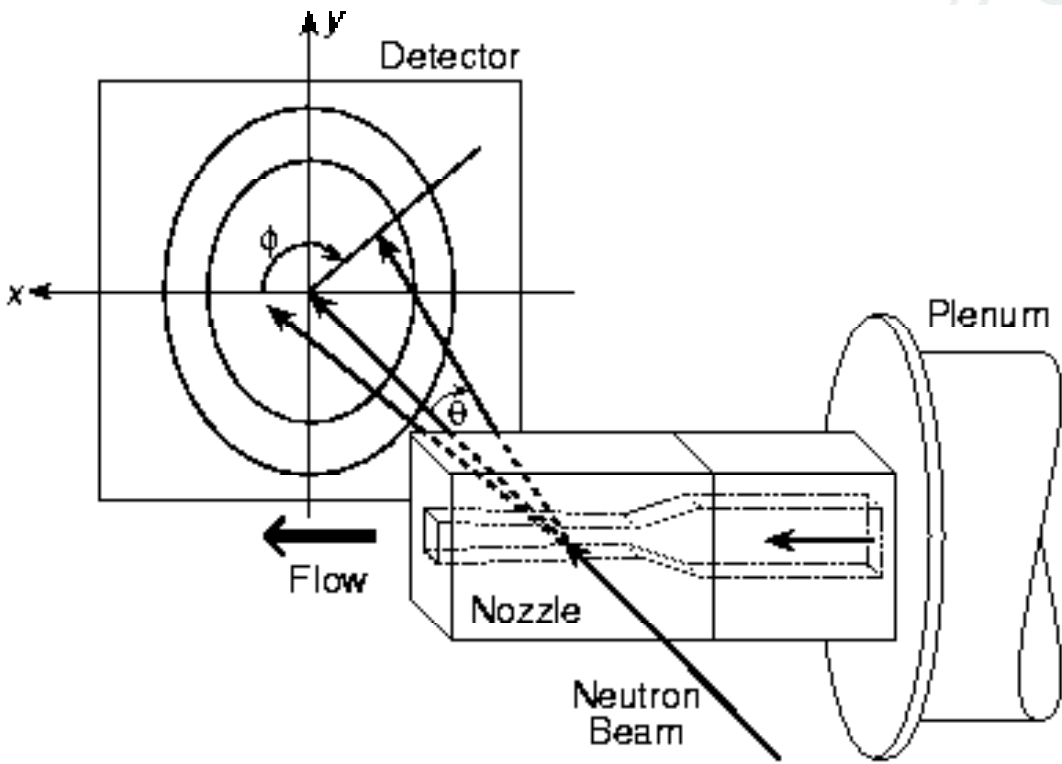


# Überschalldüse





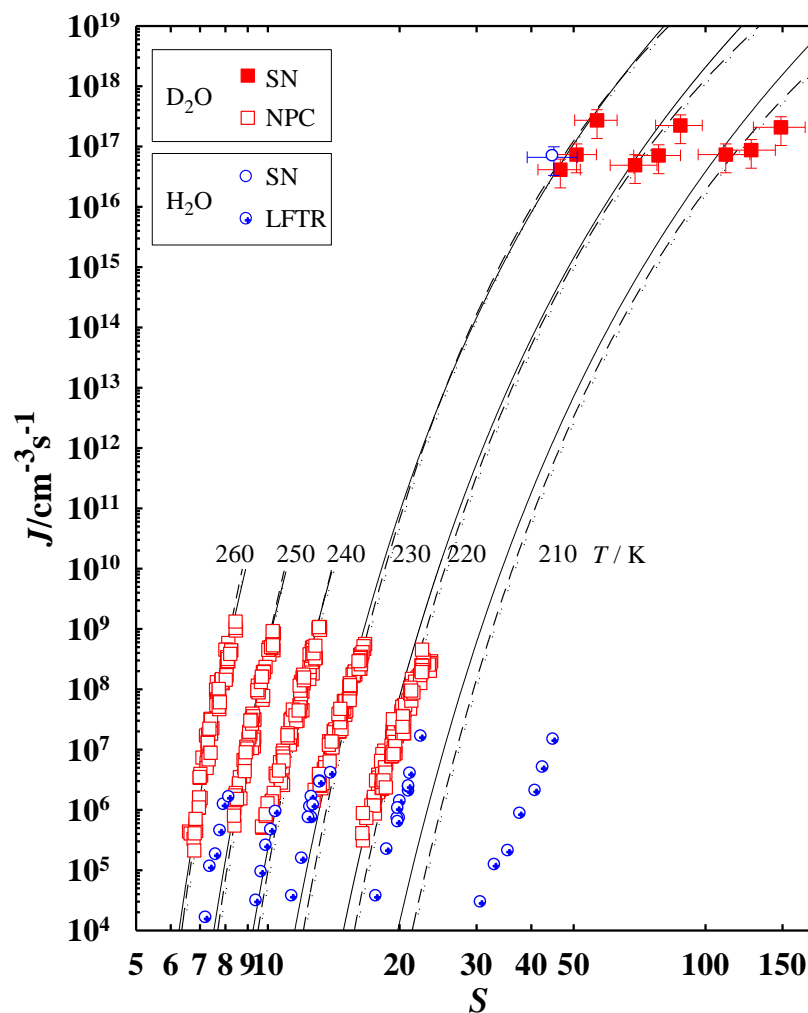
# Überschalldüse - SANS



$\theta$  : scattering angle  
 $q$  : scattering vector  
 $\lambda$  : wavelength ( $\lambda = 0.8 \text{ nm}$ )

$N \approx 2 \times 10^{11} \text{ cm}^{-3}$      $r \approx 13 \text{ nm}$

# Keimbildungsraten



# CAMS-SANS-SAXS

- Nukleationspulschammer und Überschalldüse liefern Keimbildungsraten
- Verwendete Streumethoden:

## CAMS:

- 200 nm - 3  $\mu\text{m}$
- geringe Anzahldichte

## SANS:

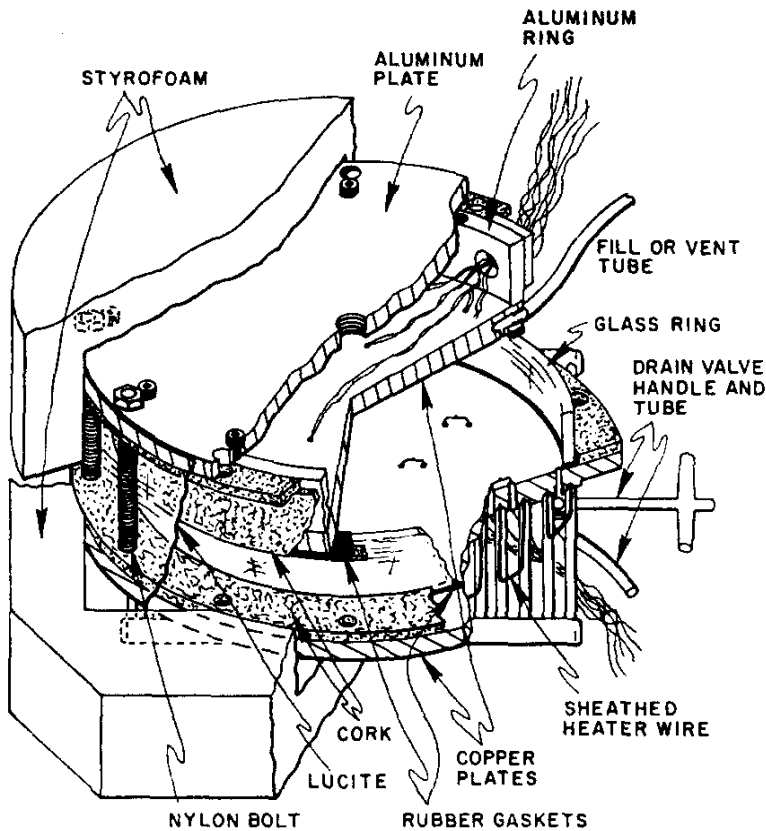
- 1-200 nm
- hohe Anzahldichte
- Kontrastierung möglich

## SAXS:

- 0.1-50 nm
- hohe Anzahldichte
- hohe Intensität
- exakte Wellenlänge
- APS:  
Punktgeometrie



# Diffusionsnebelkammer



4 STRANDS OF NICHROME HEATER WIRE ON OUTSIDE OF GLASS RING OMITTED FOR CLARITY.

FIG. 2. Cutaway view of diffusion cloud chamber.

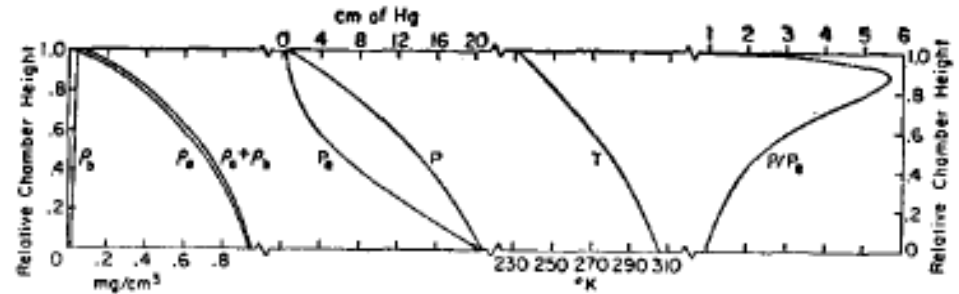


FIG. 1. Variation with height of the mass densities of the carrier gas  $\rho_b$  and the vapor  $\rho_a$ , the equilibrium vapor pressure  $P_e$ , the partial pressure of the vapor  $P$ , the temperature  $T$ , and the supersaturation  $P/P_e$ . Illustrated with data from hexane point 6.

# single piston exp. chamber

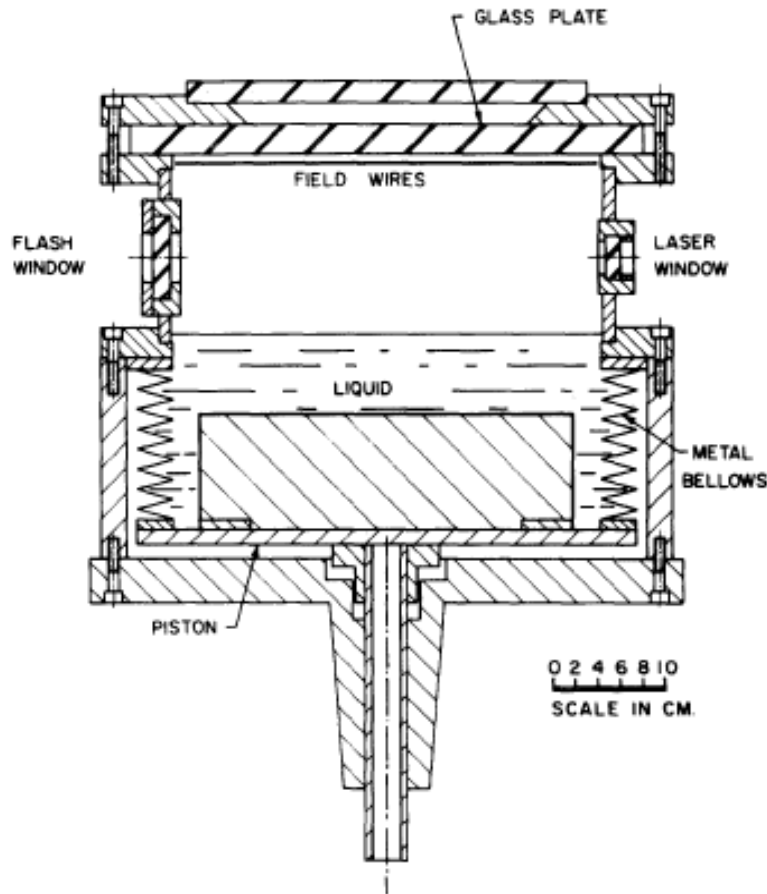


FIG. 1. Schematic diagram of the cloud chamber.

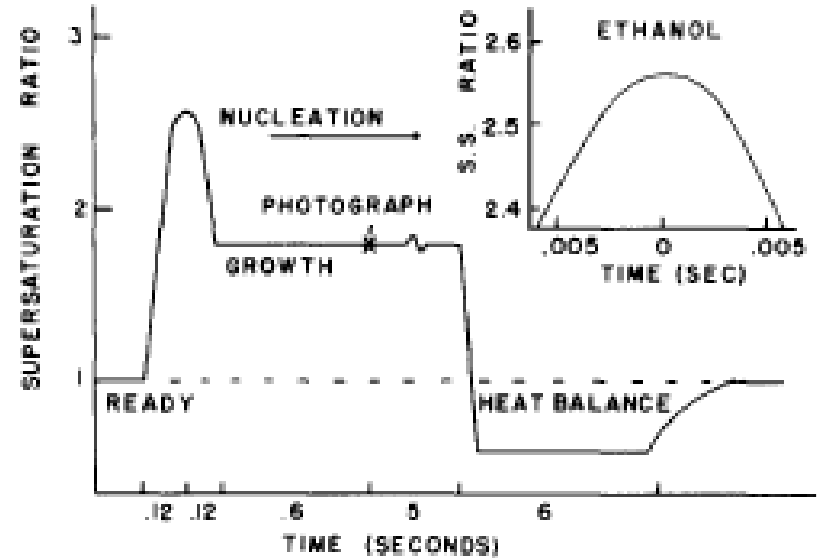


FIG. 2. Typical chamber expansion cycle for ethanol. The inset graph shows the shape of the nucleation pulse. See the text for an explicit expression for the pulse as a function of time and pressure.



# piston expansion wave tube

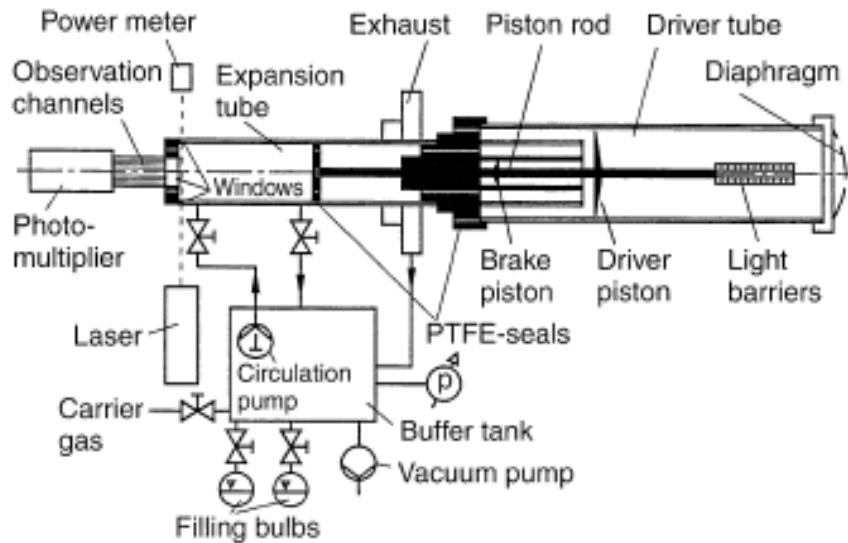
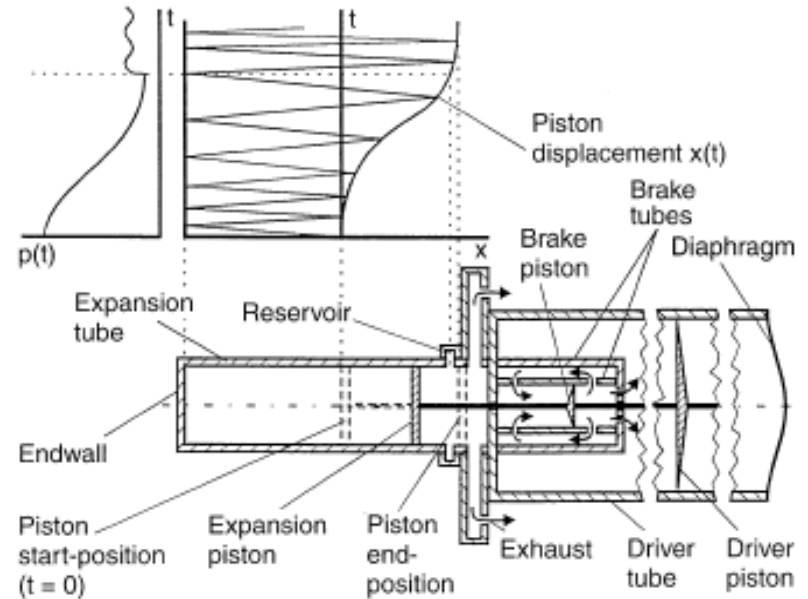
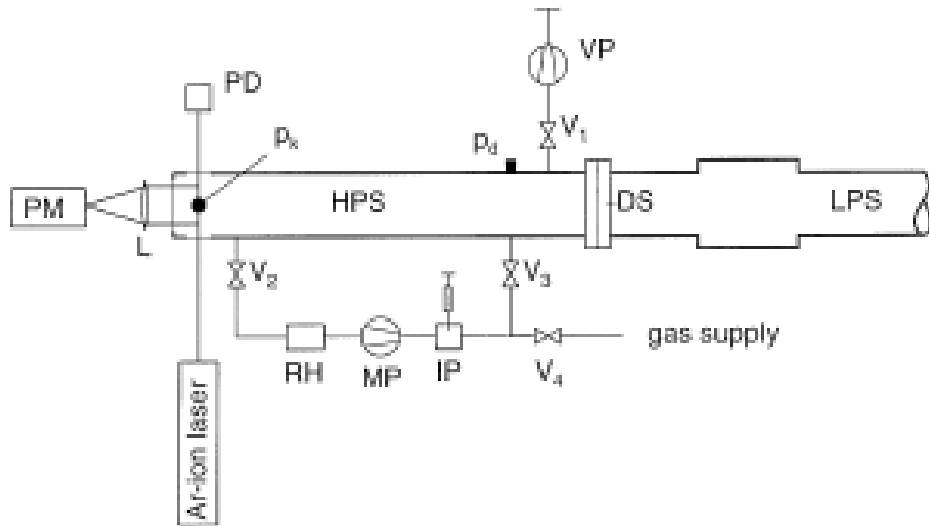


Fig. 2. Schematic of the pex-tube set-up



# pulse expansion wave tube



HPS	high pressure section	$V_{1, \dots, 4}$	valves
LPS	low pressure section	VP	vacuum pump
DS	diaphragm section	MP	mixing pump
PM	photomultiplier	IP	injection point
PD	photodiode	$p_d$	static pressure (Druck PDCR 200)
L	lens	$p_x$	dynamic pressure (Kistler 603 B)
RH	humicap (HMP 124B)		

Fig. 6. Pulse-expansion wave tube setup

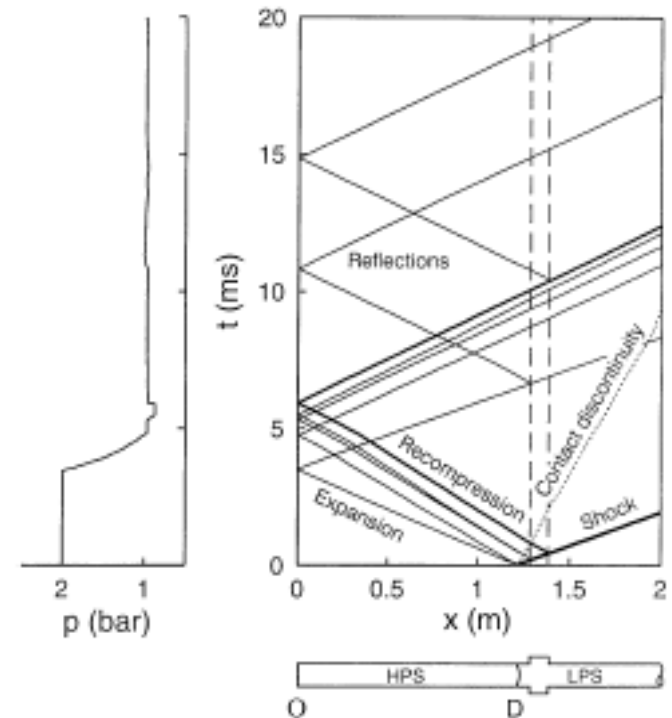


Fig. 2.  $x-t$  diagram of the wave pattern in the pulse-expansion wave tube with schematic tube configuration and pressure versus time diagram. The dashed lines show the position of the local widening. A nucleation pulse is formed at the observation point O at the endwall of the HPS. D denotes the position of the diaphragm. The diagrams were calculated with the Random Choice numerical Method (RCM). The calculations were done with nitrogen gas, with pressures of 2 bar in the HPS, and 1 bar in the LPS

# pulse expansion wave tube

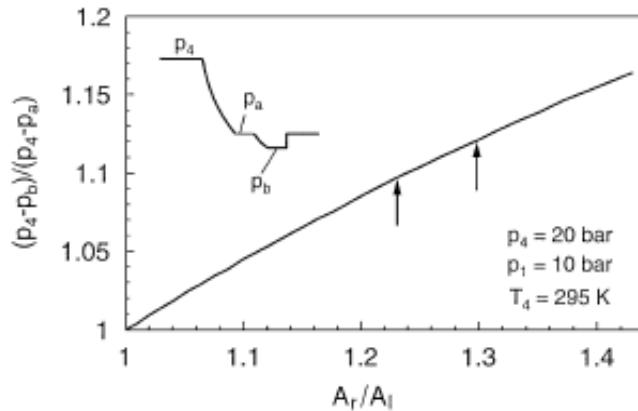


Fig. 4. Depth of the nucleation pulse compared to the initial expansion, as a function of the ratio of the cross-sectional areas ( $A_r$ ) and ( $A_l$ ). The initial conditions of the HPS are listed in the figure. The gas was methane in both the HPS and LPS. The arrows indicate the ratios of the cross-sectional areas used in the nucleation experiments

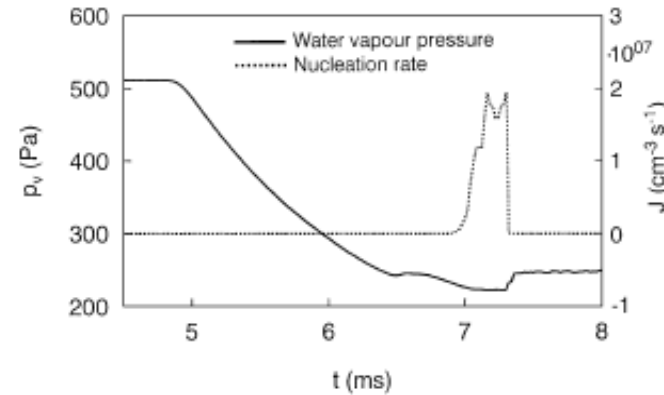
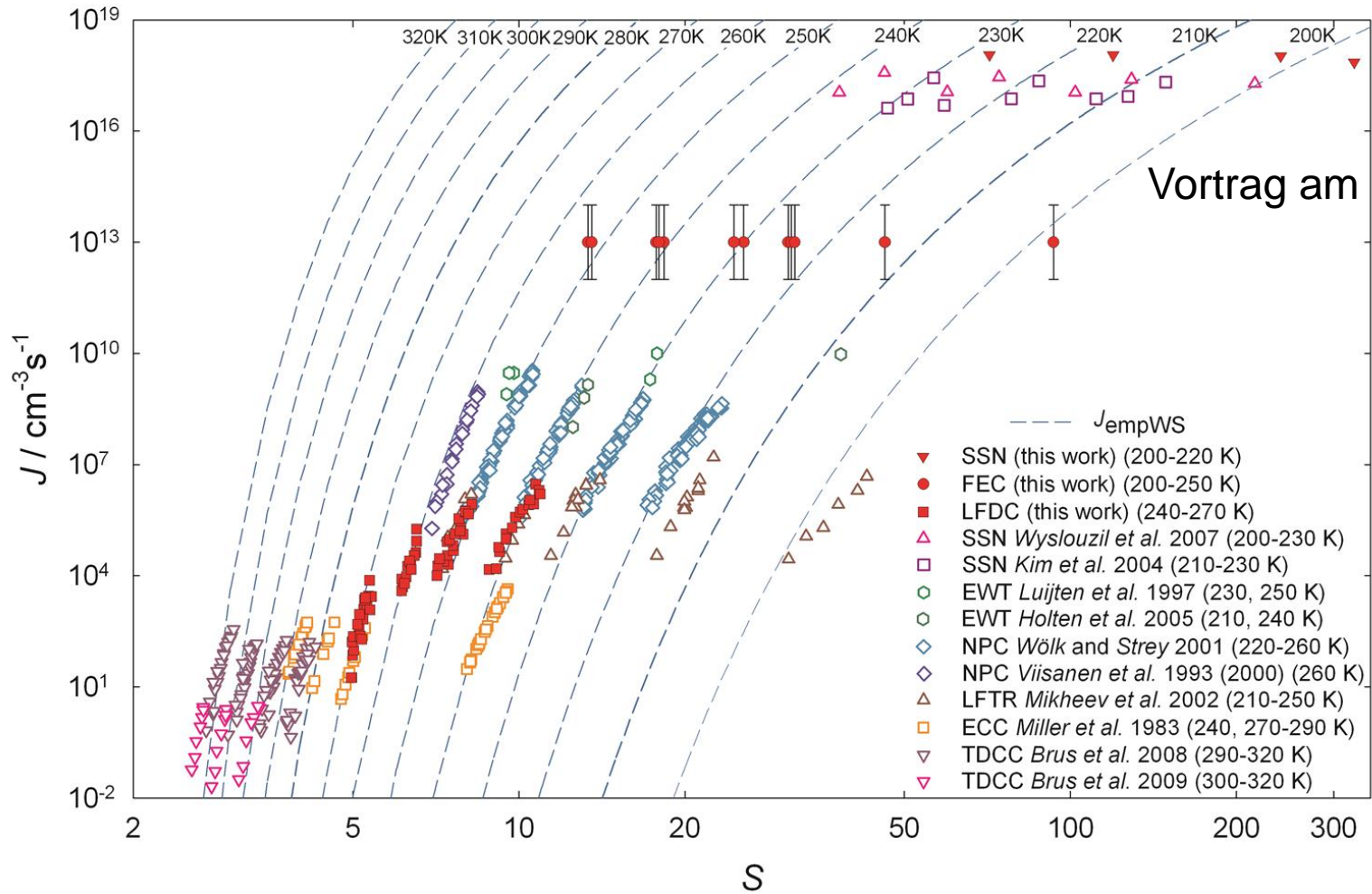


Fig. 5. Simulation of the nucleation rate by classical nucleation theory for water vapour during the nucleation pulse. The pressure history is taken from an experimental run

# Keimbildungsraten

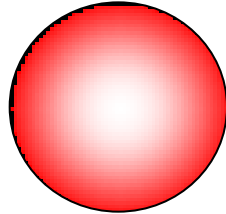


Vortrag am Donnerstag



# Theorie

classical  
nucleation  
theory



$$J = K \exp\left\{-\frac{\Delta G^*}{kT}\right\}$$

$K$  = kinetischer Vorfaktor

$\Delta G^*$  = Keimbildungsarbeit

## Annahmen:

1. sphärisch, inkompressibel, homogen mit scharfer Grenzfläche
2. Dampfphase verhält sich ideal
3. Flüssigkeit hat die gleiche thermodynamischen Eigenschaften (Grenzflächenspannung, Dichte, Dampfdruck) wie eine flache bulk-Phase





# Theorie - CNT

$$\Delta G(n) = G(n) - G(0)$$

$G(n)$  freie Gibb's - Enthalpie eines Systems bestehend aus Dampfphase und einem Tropfen der Größe  $n$

$G(0)$  freie Gibb's - Enthalpie der reinen Dampfphase

Von der Thermodynamik wissen wir, dass die freie Gibb's – Enthalpie gegeben ist mit:

$$G(p, T, N) = U - TS + pV = \mu N$$

Für die reine Dampfphase gilt somit:

$$G(0) = \mu_v N$$



# Theorie - CNT

Die freie Gibb's - Enthalpie  $G(n)$  eines Systems bestehend aus einer Dampfphase und einem Tropfen der Größe  $n$  ist gegeben durch:

$$G(n) = \mu_v N_v + \mu_l n + \sigma A$$

mit  $\Delta G(n) = G(n) - G(0)$

$$\begin{aligned} \Rightarrow \Delta G(n) &= \mu_v N_v + \mu_l n + \sigma A - \mu_v N & N = N_v + n \\ &= \mu_v (N - n) + \mu_l n + \sigma A - \mu_v N \\ &= -\mu_v n + \mu_l n + \sigma A \end{aligned}$$

$$\Delta G(n) = -n \Delta \mu + \sigma A \quad \Delta \mu = \mu_v - \mu_l$$



# Theorie - CNT

## Evaluierung von $\Delta\mu$ :

Gibbs-Duhem Gleichung bei konstanter Temperatur:  $Nd\mu_v = Vdp_v$

Annahme - Dampf verhält sich ideal:  $p_v = NkT / V$      $V/N = kT/p_v$

$$d\mu_v = \frac{V}{N} dp_v = kT \frac{dp_v}{p_v}$$

$$\Rightarrow \int_{\mu_v^{eq}}^{\mu_v} d\mu_v = kT \int_{p_v^{eq}}^{p_v} \frac{dp_v}{p_v}$$

$$\Rightarrow \mu_v(p_v) - \mu_v^{eq}(p_v^{eq}) = kT \ln(p_v / p_v^{eq})$$



# Theorie - CNT

Für die Flüssigkeit mit Grenzfläche ist die Gibbs-Duhem Gleichung:

$$d\mu_l = \frac{V_l}{n} dp_l = v_l dp_l$$

$v_l$  ist das molare Volumen der flüssigen Phase

Annahme: inkompressible Flüssigkeit:  $\Rightarrow v_l = \text{const.}$  wenn der Druck variiert

$$\Rightarrow \int_{\mu_l^{eq}(p_l^{eq})}^{\mu_l(p_v)} d\mu_l = v_l \int_{p_l^{eq}}^{p_v} dp_l \Rightarrow \mu_l(p_v) - \mu_l^{eq}(p_v^{eq}) = v_l (p_v - p_l^{eq})$$



# Theorie - CNT

Im Gleichgewicht :

$$p_{eq} = p_v^{eq} = p_l^{eq}$$

$$\Delta\mu = \mu_v - \mu_l = kt \ln(p_v / p_{eq}) - v_l (p_v - p_{eq}) \quad S = \frac{p_v}{p_{eq}}$$

$$\Rightarrow \Delta\mu = kt \ln S - v_l p_{eq} (S - 1)$$

$v_l p_{eq} (S - 1)$  Volumenterm, typischerweise sehr klein  
 $\Rightarrow$  vernachlässigbar

$$\boxed{\Rightarrow \Delta\mu = kt \ln S}$$





# Theorie - CNT

Volumen eines inkompressiblen, flüssigen und sphärischen Tropfens:

$$V_l = nv_l = \frac{4\pi}{3} r_n^3$$

$r_n$  = Radius eines Tropfens der Größe  $n$

Oberfläche des Tropfens:

$$A = 4\pi r_n^2$$

$$\Delta G(n) = -n\Delta\mu + \sigma A = -\frac{4\pi r_n^3}{3v_l} kT \ln S + \sigma 4\pi r_n^2$$



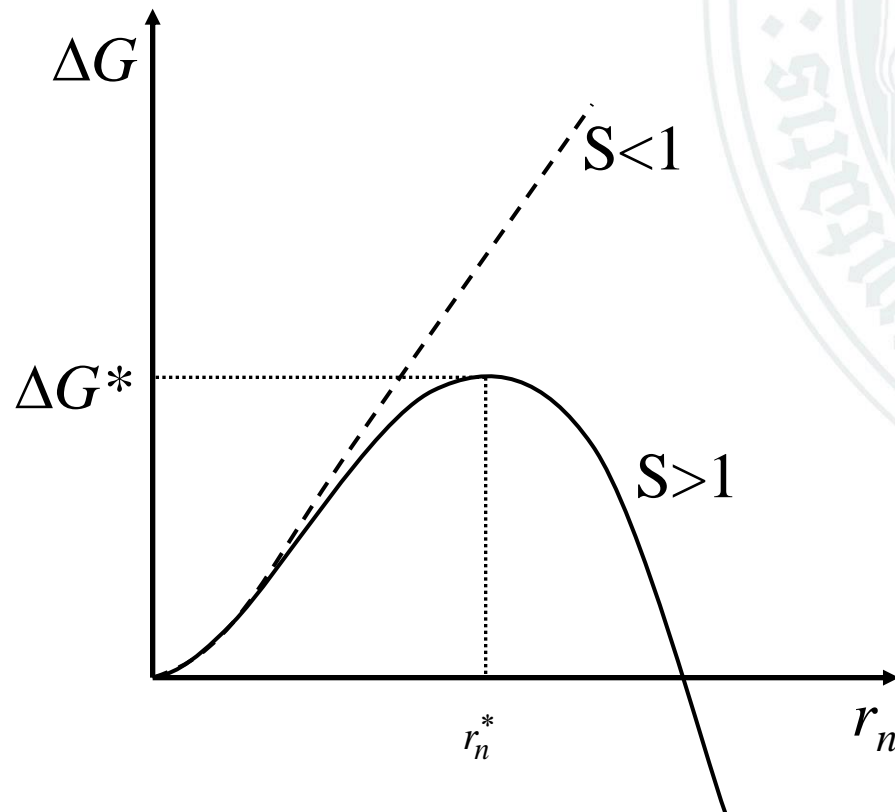
# Theorie - CNT

$$\Delta G(n) = -n\Delta\mu + \sigma A = -\frac{4\pi r_n^3}{3v_l} kT \ln S + \sigma 4\pi r_n^2$$

Für den kritischen Radius:

$$\left. \frac{\partial \Delta G}{\partial r} \right|_{r_n=r_n^*} = 0$$

$$r_n^* = \frac{2\sigma v_l}{kT \ln S}$$



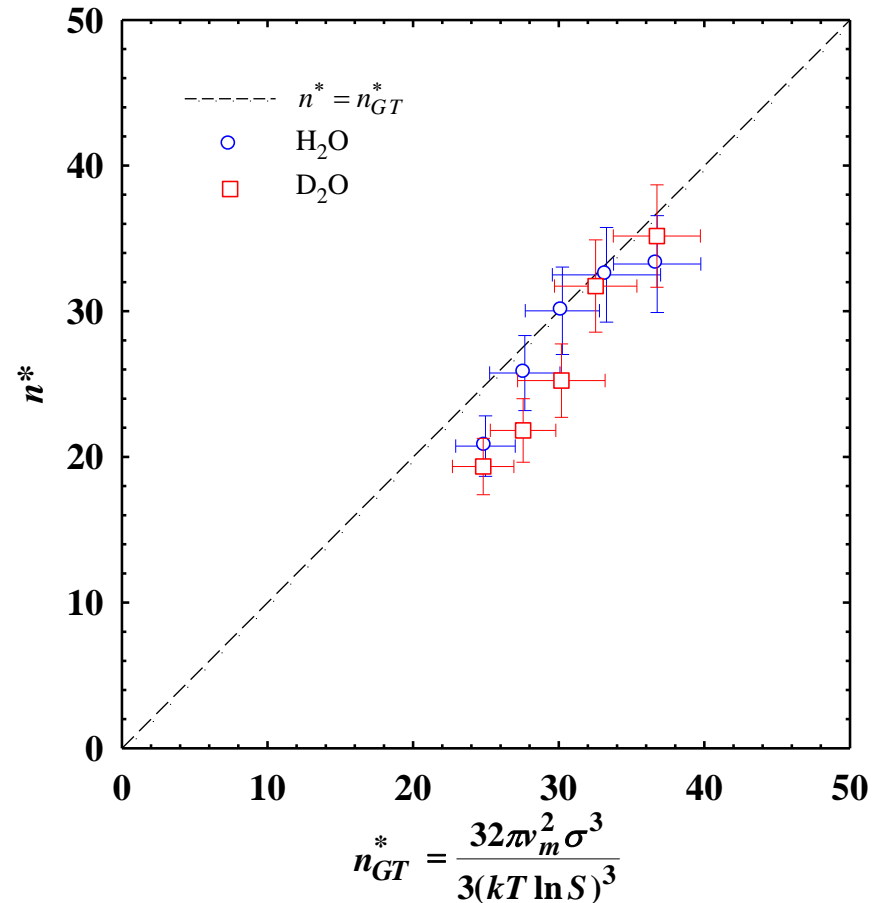
# kritische Clustergröße

## Test der Gibbs-Thomson Gleichung

$$r_n^* = \frac{2\sigma v_l}{kT \ln S}$$

$$n = \frac{4\pi r^3}{3v_l}$$

$$n_{GT}^* = \frac{32\pi v_l^2 \sigma^3}{3(kT \ln S)^3}$$



# Theorie - CNT

$$\Delta G(n) = -n\Delta\mu + \sigma A = -\frac{4\pi r_n^3}{3v_l} kT \ln S + \sigma 4\pi r_n^2$$

$$r_n^* = \frac{2\sigma v_l}{kT \ln S} \quad \Delta G^* = \frac{16\pi v_l^2 \sigma^3}{3(kT \ln S)^2}$$

$$J_{CNT} = K \exp\left(\frac{-\Delta G^*}{kT}\right) = K \exp\left(\frac{-16\pi v_l^2 \sigma^3}{3(kT)^3 (\ln S)^2}\right)$$



# Theorie

- **Classical nucleation theory [Becker and Döring 1935]:**

$$J_{CNT} = \sqrt{\frac{2\sigma}{\pi m}} v_l \left( \frac{p_v}{kT} \right)^2 \exp \left\{ - \frac{16\pi v_l^2 \sigma^3}{3(kT)^3 (\ln S)^2} \right\}$$

- **Self consistent theory [Girshick and Chiu 1990]:**

$$\Delta G_{GC} = -(n-1)\Delta\mu + \sigma A_n - \Theta \quad \Theta = \frac{(36\pi)^{1/3} v_l^{2/3} \sigma}{kT} \quad J_{GC} = J_{CNT} \frac{1}{S} \exp(\Theta)$$

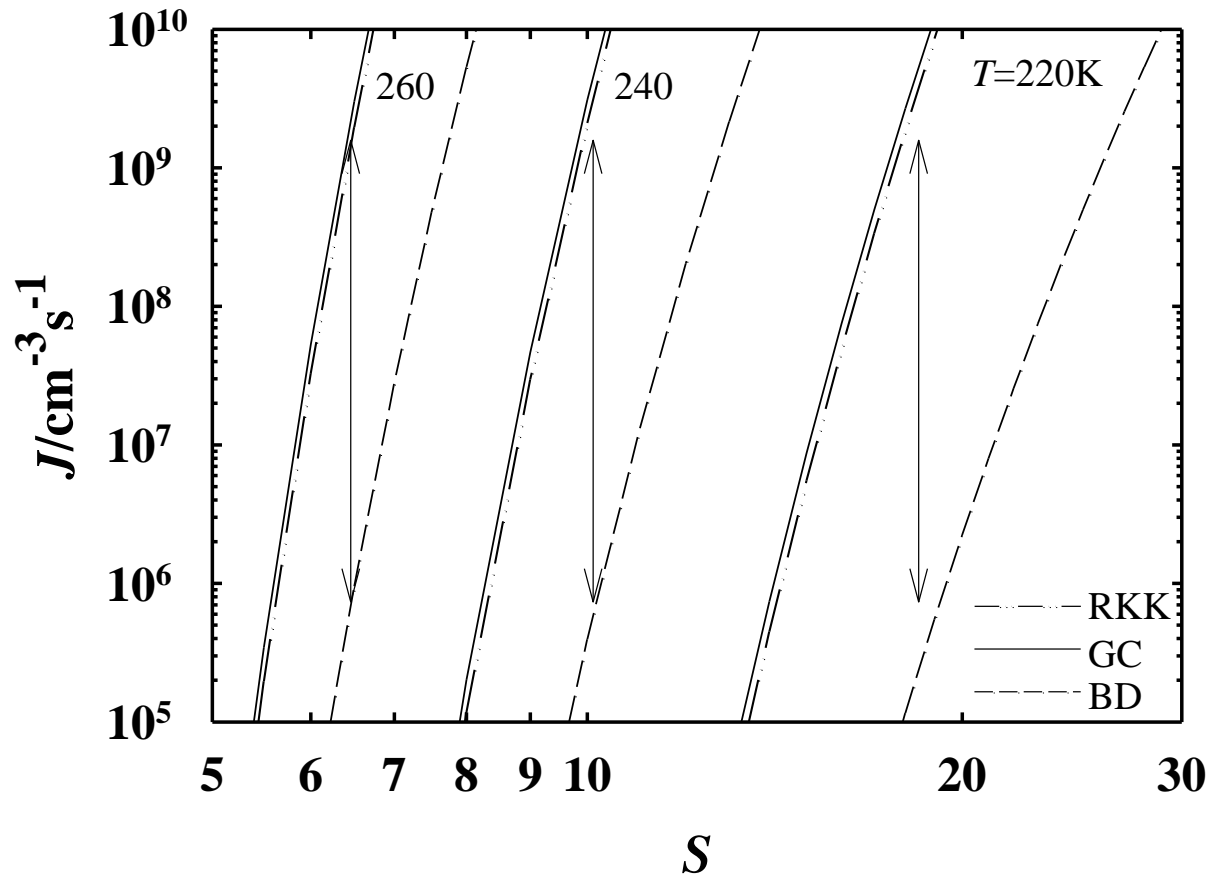
- **Reiss-Kegel-Katz theory [Reiss, Kegel and Katz 1997]:**

$$J_{RKK} = \frac{R_e}{S} J_{BD} = \frac{1}{S} \sqrt{\frac{v_l}{kT\kappa n}} \exp\{\Theta\} J_{CNT} = \sqrt{\frac{v_l}{kT\kappa n}} J_{GC}$$

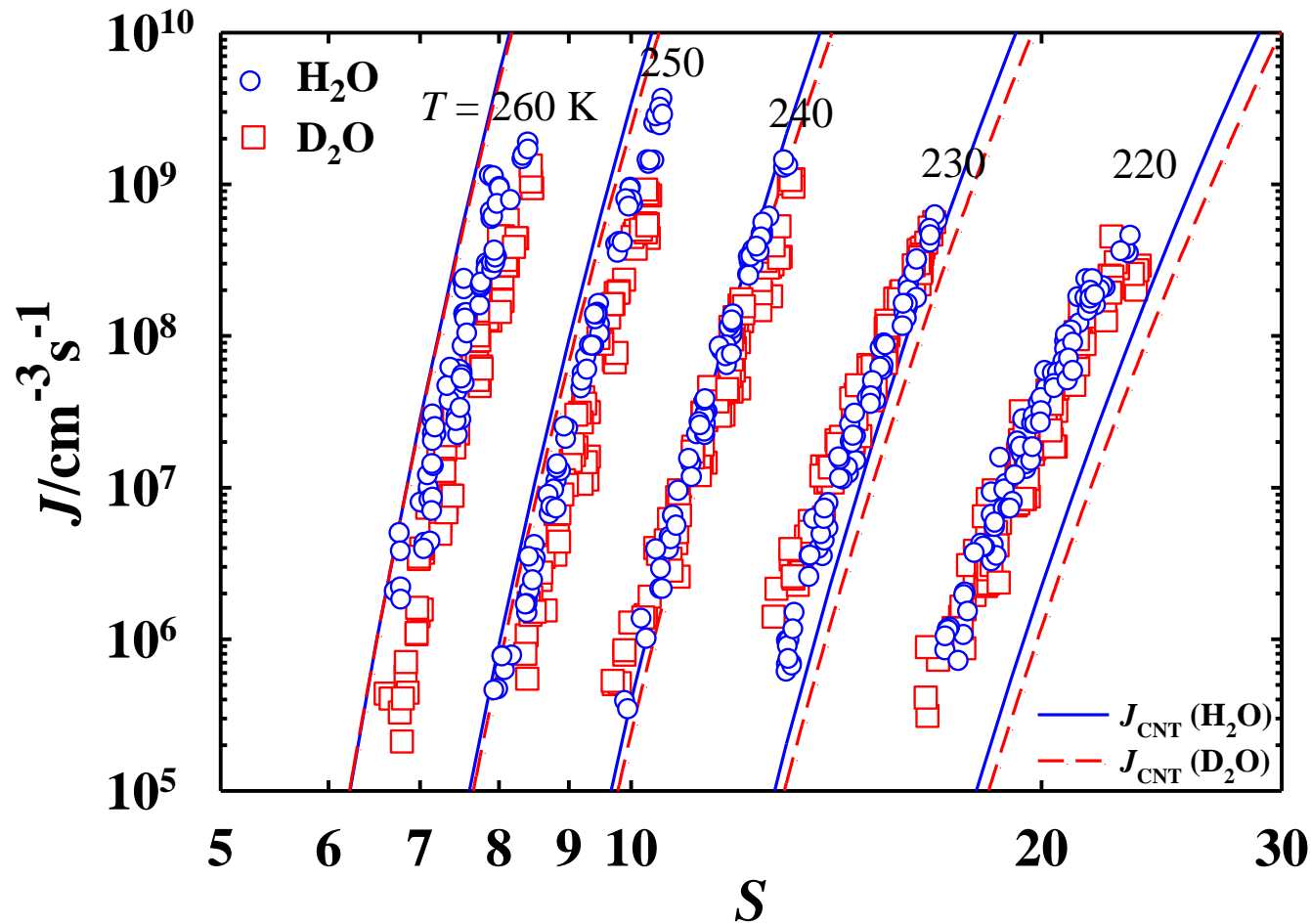




# Theorie



# Keimbildungsraten / Wasser



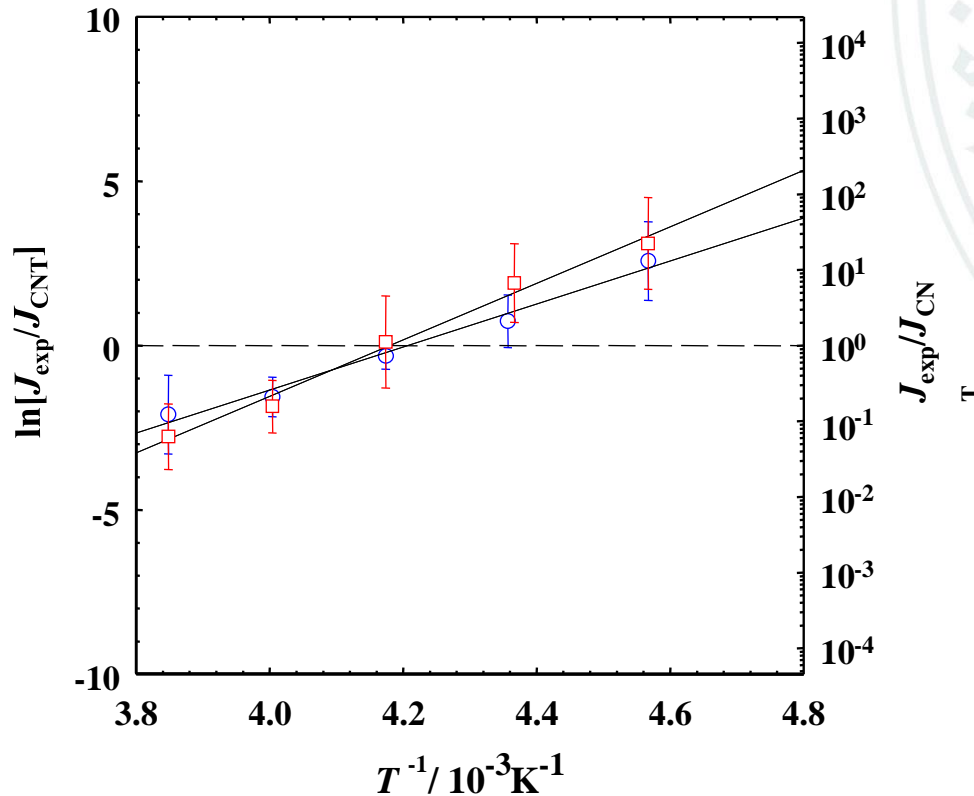
# Analyse

Theorie:  $J_{\text{CNT}} = K_{\text{CNT}} \exp(-\Delta G_{\text{CNT}}^*/kT)$

Experiment:  $J_{\text{exp}} = K_{\text{exp}} \exp(-\Delta G_{\text{exp}}^*/kT)$

$$\ln \frac{J_{\text{exp}}}{J_{\text{CNT}}} = \ln \frac{K_{\text{exp}}}{K_{\text{CNT}}} + \frac{\Delta G_{\text{CNT}}^* - \Delta G_{\text{exp}}^*}{kT}$$

$$\ln \frac{J_{\text{exp}}}{J_{\text{CNT}}} = A + B \frac{1}{T}$$



○ H<sub>2</sub>O:

$$A = -27.56$$

$$B = 6.5 \cdot 10^3 \text{ K}$$

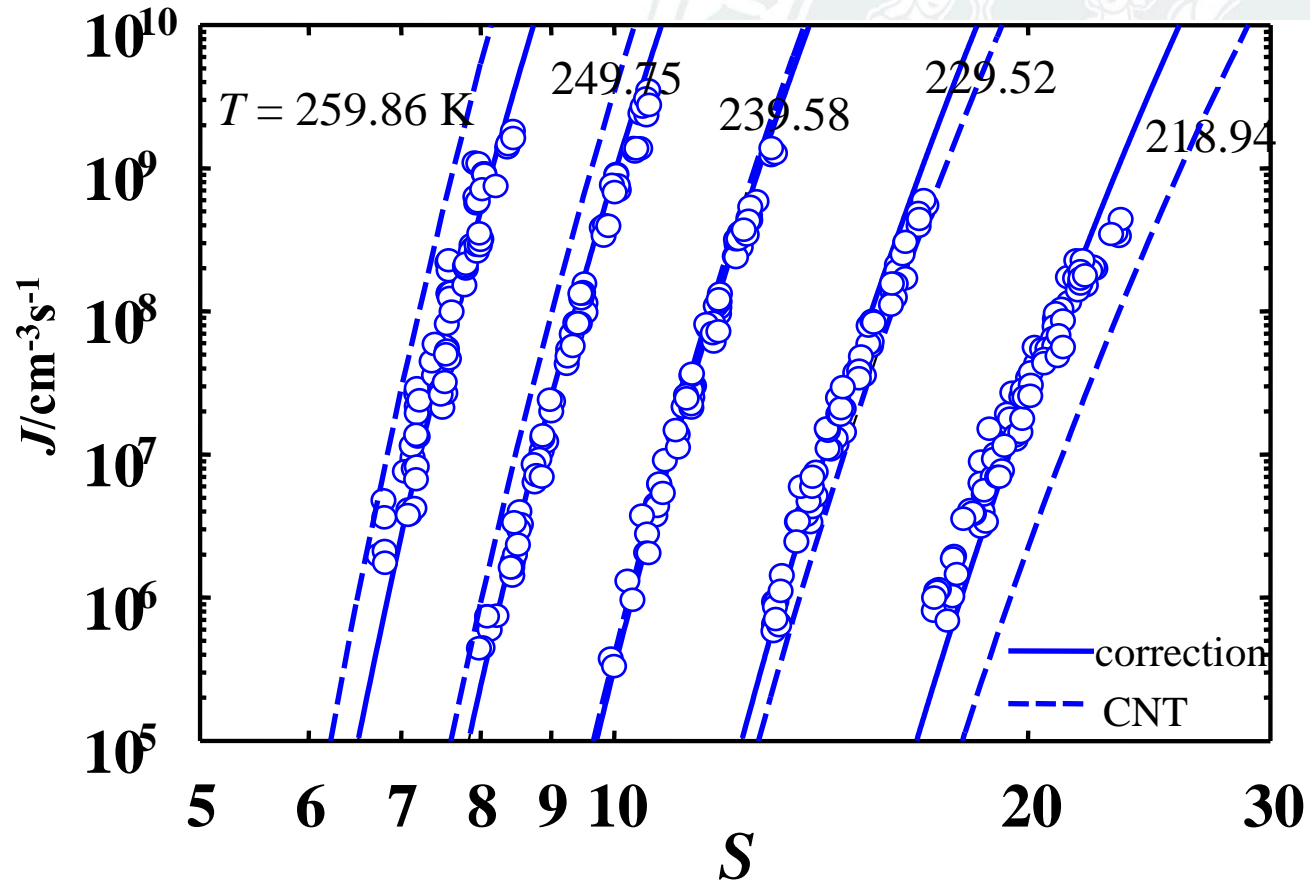
□ D<sub>2</sub>O:

$$A = -35.98$$

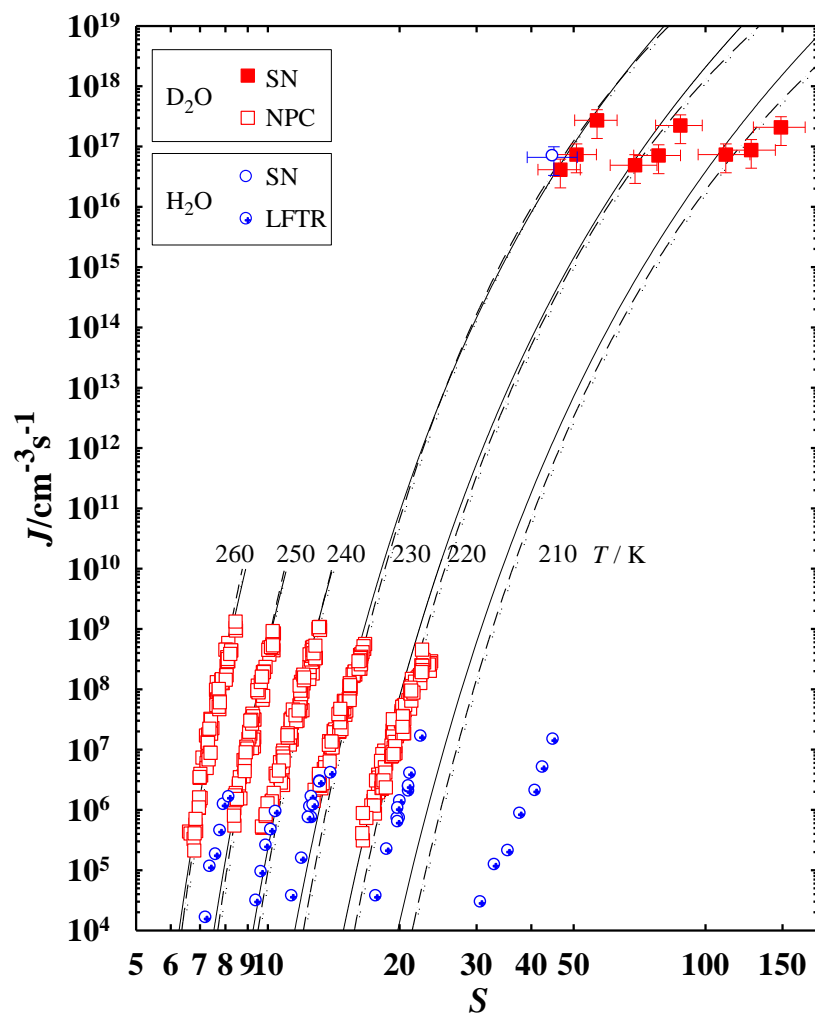
$$B = 8.6 \cdot 10^3 \text{ K}$$



# empirische Korrelation



# Keimbildungsraten



—  $J_{\text{corr}}$

